Optimal Patterns of Growth and Aid
The Case of Pakistan

by
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One of the principal means for poor countries to accelerate their development is by using external resources to supply additional imports and to finance a higher level of investment. While this policy offers substantial benefits, it also requires that the structure of the economy be adapted to accommodate the expected resource inflow over a substantial period of time. For this reason, the extent of reliance on external capital—public and private—becomes one of the critical elements of development strategy.

There has been relatively little theoretical study of the benefits and costs of using a controlled inflow of resources to promote development. Formal growth models typically either ignore this variable or take it as fixed. In the formulation of development programmes by planning organizations, the projected inflow of aid and private capital is determined largely on a historical and political basis rather than through a systematic evaluation of alternatives. This is true in Pakistan as well as in most countries receiving foreign assistance.

This paper explores the properties of optimal growth strategies in which the total amount and time pattern of the resource inflow can be varied within limits. The problem is studied both from the point of view of the borrowing country trying to make the best use of its domestic and foreign resources and from that of the lender trying to assess the productivity of additional amounts of public assistance in different recipient countries. These different viewpoints are reflected in alternative forms of the objective function which is maximized to determine the optimal policy.

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The linear programming model used in this study was suggested by Robert Dorfman, who took the lead in its initial formulation. It is derived from the projection model used in Chenery and Strout [4]. We are indebted to Robert Dorfman, Alan Strout and Joel Bergsman for helpful comments.
While the formulation of the problem is designed to bring out its general features, the resulting programming model is applied to the planning situation described in the Pakistan Perspective Plan for 1965-1985. Apart from the variables affecting the inflow of external capital—which is taken as given in the Pakistan Plan—we have taken most of our other assumptions from the Plan in order to isolate the effects of variation in external resources. Generalization of the results to other countries will be attempted in a subsequent paper.

I. THE ANALYTICAL FRAMEWORK

A. Growth Models and Planning Models

The problem of optimal growth paths over time has been studied only under assumptions which are rather far from those describing the policy choices facing underdeveloped countries. The main weaknesses of existing growth models are that they i) assume closed economies, ii) focus mainly on the allocation of resources between investment goods and consumer goods production, iii) ignore some of the central constraints on policy, and iv) study long-run equilibrium conditions rather than developments over a relevant planning period. There is therefore little carryover of the specific results so far achieved by formal analysis to the problem at hand. Their principal contribution is to show the importance of formulating an explicit welfare function and of relating alternative strategies to both the parameters in this function and the restrictions placed on the system.

The analytical framework used here is largely derived from detailed empirical models of open economies which are dependent on external assistance. These studies utilize models in which import choices and alternative levels of external capital are explicitly considered. Multisectoral analyses are used in most of them to derive relations among capital inflow, import requirements, savings rates, investment allocation and overall growth for the planning period considered. In these disaggregated open models, the balance of payments limit replaces the capacity to produce investment goods as a general factor limiting growth. The inflow of external capital plays the dual role of raising both this specific resource limit and the savings limit on the rate of investment.

While most of the planning models cited above include some elements of optimization for a five- or ten-year period, they do not consider the pattern of capital inflow over a long enough period to show the welfare implications of alternative strategies of aid and growth. However, they do suggest that there are

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1 Despite these differences, we have gotten considerable insight into the present problem from Goodwin [5], Uzawa [13], and Stoleru [11].

some common characteristics of optimal patterns of aid and growth. The model
developed for the present analysis is designed to explore this possibility more
systematically. It tries to relate the features of optimal growth patterns to the
development policies and objectives of aid recipients and the assistance policies
of the donors.

B. The Empirical Background

Pakistan has been chosen for this study because it receives substantial exter-
nal aid and it has an explicitly formulated twenty-year plan for its future growth.
It is also broadly representative of the initial conditions from which many poor
countries are attempting to start a process of accelerated growth. The typical
features most relevant to the present analysis, taken from a comparative analysis
of thirty-one underdeveloped countries [4], are summarized below. Compared
to the medians of this sample, Pakistan starts from a generally less favourable
position of per capita income, savings and investment rates, and previous growth
of GNP, but the improvement in its development performance in recent years
has been significantly better than the average.\(^3\)

The following observations provide a basis for both the design of the model
in Section II and the range of values for the policy variables over which it will be
tested in Section III.

1) External resources—three quarters of which are classified as public
assistance—normally finance 20 to 30 per cent of both investment and imports
in underdeveloped countries and a higher proportion of the increases in these
elements in typical cases of rapid development.

2) There is substantial evidence of a limit to the ability of developing coun-
dtries to transform large increases in external resources into productive investment.
The most convenient measure of this absorptive capacity limit is the rate of in-
crease in investment which a country can achieve on a sustained basis. Rates
of 15 to 20 per cent per year have been observed in a number of countries, but
there has been no case of a higher value over any substantial period.

3) Shortages of imported investment goods and raw materials provide a limit
to growth in a number of countries. In contrast, while the capacity to produce
the non-importable components of investment is a potential bottleneck, it is more
easily avoided and rarely observed.

4) Gross marginal savings rates are significantly above the initial average
rates of about 12 per cent of GNP in the 31-country sample, they reach 30 per cent

\(^3\) The initial conditions and measures of recent performance in Pakistan are given in Section
III.
in the upper quartile of countries. However, there are no observed cases of marginal rates approaching the levels of 50 per cent or more which are implied by most theoretical analyses of the "optimal" rate of saving.

5) The availability of external capital permits an economy to grow at the limit corresponding to its ability to increase its capital stock rather than at the lower rate implied by its ability to increase domestic savings. A period of accelerating growth in which investment, savings and external assistance all increase is therefore to be expected; it is observed in a number of countries.

6) Under present institutional arrangements for the transfer of resources from advanced to developing countries, the amount available is rationed among claimants whose total demands substantially exceed the supply. Since supply conditions vary greatly among recipients, however, different formulations of the restriction on external capital may be appropriate for different countries.

C. Elements of a Two-Sector Model of An Open Economy

These empirical observations require a substantial reformulation of conventional aggregate growth models. It remains to be seen whether some of the qualitative results of two-sector closed-economy analysis can be carried over to the open-economy case. However, with the addition of the choice of the capital inflow over time, the optimizing problem can be put in similar terms. In both cases, we are primarily interested in the general behaviour of the principal variables which describe a growth pattern or development strategy over time. The empirical studies summarised above suggest the following characteristics for a two-sector model of an open economy.

Sector Breakdown: The basis for disaggregating the economy is crucial because of the limits which it imposes on the possibilities for future growth and of the way in which it reflects the role of the capital inflow. Disaggregation into two sectors should show the capacity of an open economy to transform domestic resources into foreign exchange, which can then be used to fill in the gaps between the composition of demand and the composition of supply. While the foreign exchange bottleneck cannot be identified with a particular industrial category, the need to allocate capital and labour to increasing its supply is quite similar to the allocation of resources to the production of investment goods in a closed economy. In our model, a category of "trade-improving" production will be identified, which produces either increased exports or substitutes for goods presently imported. Whether the corresponding commodity is cotton, steel or machinery is irrelevant.

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4 See for example Goodwin [5] and Stoleru [11] who have derived illustrative paths of optimal savings for underdeveloped countries from a variety of assumed welfare functions.

5 A similar conceptual problem arises in identifying investment goods in a two-sector model, which can only be solved empirically by means of an interindustry analysis.
A two-sector model which embodies this distinction could be derived from solutions to an interindustry model in which the input structure and composition of final demand is fixed. In this way, the inputs of capital and imported goods required for an expansion of output with the existing economic structure could be determined. Possibilities for import substitution or introduction of new exports could then be described by additional activities as in [2]; the possibilities of transforming capital and labour into foreign exchange ("trade improvement") could then be determined by an optimizing procedure. In a multisectoral analysis the result would be a rise in the incremental capital cost as the output of the trade improvement sector rises, reflecting the operation of the principle of comparative advantage. (For Pakistan, we will represent this input function by a constant incremental capital coefficient, since we cannot estimate the function directly.)

Scarc Factors: Instead of capital and labour, the scarce factors relevant to our analysis are capital and foreign exchange. The rationing of external capital means that its supply must either be taken as given or valued at an opportunity cost which reflects its scarcity. The transformation of unskilled into skilled labour can be treated as part of the investment process, however, and total labour supply is not likely to be a limiting factor within the period relevant for the analysis.

Policy Objectives and Restrictions: Within the limitations of two-sector analysis, it is desirable to incorporate restrictions which reflect both the limited flexibility of economic systems and the political limits to feasible policy changes. For example, any significant reduction in per capita consumption (which occurs in many so-called "optimal" growth paths) should probably be ruled out as politically infeasible. The introduction of such constraints makes the conclusions more realistic, although the results are less susceptible to generalization in form of simple decision rules.

II. THE MODEL

The problem of determining an optimum pattern of aid and growth over time will now be stated in linear programming form. The objective is to maximize a social welfare function, incorporating benefits (consumption) and costs (capital inflow) for each period of time. The constraints are the policy goals and the definitional, structural and behavioural relationships for each time period. Variables and parameters are defined in Table I. The variable and parameter values used in the Basic Solution are given in Tables II, III and IV. All these tables appear at the end of the text.

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6 The unemployment rate in Pakistan is estimated in the Pakistan Plan [6] at 20 per cent and the growth of population at 2.6 per cent. In countries having less unemployment, a more explicit treatment of the potential labour limitations might be needed, as in Chenery and Bruno [3].
The Welfare Function

We wish to maximize the general welfare function

\[ W = \sum_{t=1}^{T} \frac{C_t}{(1+i)^t} + \eta \sum_{t=1}^{T} \frac{V_t}{(1+i)^t} - \sum_{t=1}^{T} \frac{F_t}{(1+i)^t} \]

where

\[ \eta = \delta \sum_{t=1}^{\infty} \frac{(1+r)^t}{(1+r)^T + t} \]

This function has three parts: 1) the discounted sum of consumption prior to the terminal year of the plan; 2) an indicator of the discounted value of consumption in all years posterior to the plan, with a variable weight \( \eta \); 3) the discounted sum of total capital inflow with a weight \( \gamma \), representing the price of foreign capital, which varies according to the supply conditions for the country concerned.

By varying \( \gamma \) and certain policy constraints it is possible to simulate a wide range of supply conditions. If no policy constraints affecting supply conditions were added, the supply of foreign capital would be assumed to be infinitely elastic at the price \( \gamma \).

This assumption of infinite elasticity is modified in the two alternative forms of the model in order to yield a more realistic statement of the scarcity of foreign capital.

1) In the "Basic Solution" defined below we have imposed the condition that foreign aid must terminate in a given year \((T-n)\) prior to the terminal year of the analysis \((T)\). In this case the supply of foreign capital remains perfectly elastic at the price \( \gamma \) prior to \((T-n)\), but for years after \((T-n)\) the economy must be self-sufficient.

2) In a second alternative form we assume that the total quantity of discounted aid received during the plan cannot exceed a given amount.

The results obtained by solving the model using these different specifications of the supply conditions are discussed in Section III.

The question arises as to whether the welfare function is formulated from the point of view of a recipient or a donor. The answer is that it can represent
views of either recipients or donors, as well as a variety of views within each group. Different welfare assumptions are represented by the values given to the parameters in the objective function. For example, a country having a high preference for improvement of living standards during the period of the plan compared to concern for living standards in the distant future would give a relatively low weight to post terminal consumption. This view implies a low value of $\delta$ or a plan discount rate (i) that is low relative to the post-plan discount rate (r). The higher rate in later periods can also reflect a judgement as to the diminishing marginal utility of added consumption.

A second example is the donor or planning authority which desires the recipient country to become self-sufficient by the end of the plan period. In this case conditions in the short run are not of primary concern, though certain minimum standards must be met. This view can be represented in the Basic Model by a high value of $\delta$. The donor would not view supply conditions as given, but would use the model to help in establishing supply conditions.

Our treatment of post-plan consumption in the welfare function assumes that after period T the economy will proceed along a path of self-sustaining growth and that a constant portion $(1 - \alpha)$ of income will be consumed. An estimate of the self-sustaining rate of growth ($\phi$) can thus be determined.\(^{10}\)

Our use of discount rates in the welfare function is based upon the standard time preference arguments. We allow for a higher discount rate in later years, which can be justified in terms of diminishing marginal utility of rising per capita income. As time passes there is a corresponding rise in per capita consumption and the marginal utility of consumption declines.\(^{11}\) (The discontinuity of year T is chosen for convenience but does not significantly affect the conclusions).

**Definitional Equations**

GNP is the sum of the net output of the two sectors: regular production and for trade improvement.

$$V_t = V_t^\circ + V_t^1$$

Gross investment is similarly the sum of investment in the two sectors:

$$I_t = I_t^\circ + I_t^1$$

Investment is equal to domestic savings plus net foreign capital:

$$I_t = S_t + F_t$$

\(^{10}\) As $t$ becomes large, the average savings rate approaches the marginal savings rate, and the aggregate capital-output ratio approaches a (constant) weighted average of the two sectoral capital-output ratios in inequalities (7) and (8). The ratio of the average savings rate to the aggregate capital-output ratio yields the long-run rate of self-sustaining growth.

\(^{11}\) This argument is made by Goodwin [5] in determining the optimal savings rate.
The trade gap is determined by the excess of the demand for “traditional” import over the sales of “traditional” export less the output of the trade improvement sector 12.

It must be filled by a net flow of external resources, \( F_t \).

\[
F_t = (M_t - E_t) - V_t^1.................................(4)
\]

This definition of the trade gap leads to a formulation of the national income equality which shows trade-improving production as a reduction in the trade gap:

\[
V_t = C_t + I_t + E_t - M_t + V_t^1.........................(5)
\]

Traditional exports are assumed to grow at an exogenously determined rate:

\[
E_t = E_o (1 + e)^t........................................(6)
\]

These exports can be produced at the capital-output ratio of regular production.

**Structural and Behavioural Constraints**

Since labour is assumed to be in surplus, production in each sector is limited only by capital in that sector and by the supply of imports. The structure of the economy in the base year is the basis for defining the limit to regular production:

\[
V_t^o \leq V_o + \frac{1}{k_o} \sum_{\tau=0}^{t-1} Y_{\tau}^o........................................(7)
\]

Production for trade improvement requires a higher capital-output ratio, and, by definition, investment in this sector begins only after the plan has commenced:

\[
V_t^1 \leq \frac{1}{k_1} \sum_{\tau=1}^{t-1} Y_{\tau}^o........................................(8)
\]

The aggregate capital output ratio is a weighted average of the capital-output ratios of the two sectors. It changes over time as the distribution of investment between the two sectors changes. In the period of self-sustaining growth the proportion of trade-improvement investment asymptotically approaches a limit of about 25 per cent of total investment. In the basic solution of the model, the economy is forced to self-sustaining growth after \( t = 20 \).

Maximum savings in any year is a function of base-year savings and the increase of income since the base year:

\[
S_t \leq S_o + \alpha(V_t - V_o)........................................(9)
\]

As \( V_t \) becomes large average savings rate will approach the marginal savings rate \( \alpha \). The marginal savings rate can be viewed as partially a behavioural constraint and partially an instrument of policy. Within certain limits the government could

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12 As explained above “traditional” imports and exports mean imports which would be required and exports which could be sold if the structure of the economy were to remain unchanged from the base year.
institute policies which would affect $z$. However, within the model presented here the marginal savings rate is taken as given.

The requirement for goods traditionally imported is a function of base-year imports and the increases from the base year in income and investment:

$$M_t \geq M_o + m_o (V_t - V_o) + m_1 (I_t - I_o)$$

While the marginal import rates can be affected by policy decisions, within the present model they are taken as technical parameters. The relatively high value of the marginal import rate on investment ($m_1$) produces some of the pressure of rapid growth upon the trade gap.

The observed limits to the ability of an underdeveloped country to absorb increases in the supply of capital are incorporated in the model by placing an upper limit ($\beta$) on the rate of growth of investment:

$$I_t \leq (1 + \beta) I_{t-1}$$

While an underdeveloped country may be able to raise its absorptive capacity in time, it is in the early years of the plan—when little could be done to raise the absorptive capacity—that the upper bound on growth of investment is of greatest importance.

It is also necessary for technical reasons to place a lower bound on the growth of investment. To prevent unrealistic declines in investment—which the model would otherwise yield—we have included the following constraint:

$$I_t \geq I_{t-1}$$

**Policy Constraints**

The welfare function largely defines the policy goals of the nation. However, certain goals can only be formulated in terms of absolute targets and must therefore be stated as constraints of the model. One such goal is that per capita consumption not be allowed to decline. This can be insured by the inclusion of a constraint requiring total consumption to grow at least as rapidly as population:

$$C_t \geq C_{t-1} (1 + p)$$

Another policy goal which it is necessary to formulate as a constraint is the requirement that capital inflow be terminated by some predetermined year:

$$F_t \leq 0 \text{ for } t = T-n, T$$

The significance of this modification of the aid supply condition was pointed out above in the discussion of the welfare function.

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13 The absolute limit on absorptive capacity is rather arbitrary. It implies that no further investment can take place because of shortages of complementary inputs. It would probably be more realistic to assume that above this limit further investment can be carried out but only at higher capital-output ratios and with longer time lags. It would be possible to incorporate this more realistic assumption into our linear model by using step functions.
Alternative Forms of the Model

As formulated above, the model allows the foreign assistance supply conditions to be specified in two forms: 1) the price of foreign capital (γ) may be specified, or ii) the terminal date for capital inflow (T − n) may be specified. An alteration of the model allows a third method of specifying supply conditions. In this third form we place an upper limit on the total quantity of aid received over the plan and specify neither a price of foreign aid nor a termination date. That is, we add the constraint.

\[
\sum_{t=1}^{T} \frac{F_t}{(1 + i)^t} \leq \bar{F}
\]

(15)

The three forms of the model will be discussed in Section III. It will be shown that equivalent results can be obtained from each form. For example, if a price is specified a termination date and a total quantity of aid will be endogenously determined. We can therefore summarize the three forms of the model as follows:

<table>
<thead>
<tr>
<th>Form 1</th>
<th>Form 2</th>
<th>Form 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price (γ)</td>
<td>specified</td>
<td>determined</td>
</tr>
<tr>
<td>Termination date (T − n)</td>
<td>determined</td>
<td>specified</td>
</tr>
<tr>
<td>Total aid (\bar{F})</td>
<td>determined</td>
<td>specified</td>
</tr>
</tbody>
</table>

It is, of course, possible to combine two of these forms although only one of them will turn out to be effective. This was done in the Basic Solution (see below) where both a minimum price of aid and a maximum termination date were specified. The solution then determines which limit is controlling.

Limitation of the Pattern of Aid

As explained below, foreign assistance is typically rationed by the donors on an annual basis. To reflect this supply limitation in our model, we will compute a set of solutions in which capital inflow cannot exceed a given ratio to GNP. This results in adding the following limit to the model:

\[
F_t \leq q \ V_t
\]

(16)

In the experiments discussed below \(q = .05\)

Other Limits: In developing the basic model alternative forms of some of the structural relationships were employed. The most important of these was the use of separate upper and lower bounds on the rate of growth of investment in each sector. This procedure is based on the rationale that regular production and trade improvement are actually two distinct types of investment. Trade improvement requires the construction of new plants and the development of new industries. While this assumption prevents the rapid shifting between one form of investment
and another, it does not significantly alter the qualitative form of the results. It was therefore omitted from the final form of the model.

III. GROWTH ALTERNATIVES FOR PAKISTAN

The Pakistan Planning Commission has made two twenty-year projections or "perspective plans" [6;7], as a basis for its Third Five Year Plan for 1965-1970. In both these projections the net inflow of external resources is assumed to decline steadily and to approach zero by 1985. Little reason is given for this assumption apart from the desire to become independent of foreign assistance. Its effect on other objectives of the plan, such as the terminal-year income, is not discussed.

In order to isolate the effects of varying amounts of external assistance, we start from the planning situation described by the objectives and constraints of the Pakistan Plan. The Plan document and other analyses of the Pakistan economy are used to determine plausible values for the parameters in our model and possible variations in them. We have made no attempt, however, to incorporate all of the economic and political considerations that affect the preparation of a development programme. Our results are not designed as a critique of the Plan but to suggest the possibilities for more effective development strategies if assistance policies could be modified.

Our procedure is as follows. We first determine an optimum solution to the model in its original form based on welfare objectives and performance characteristics similar to those in the Pakistan Plan. This Basic Solution provides a point of departure for several sets of experiments. The first is designed to show the welfare effects of supplying assistance under conditions that more closely approximate the present arrangement. The second set of experiments shows the effects of development performance on aid requirements. In both cases, we have assumed a range of values for the external capital inflow to show the increases in consumption and income made possible by increasing aid. Taken together, these experiments bring out the inter-relations between development strategy and foreign assistance policy and suggest the advantages of greater coordination between the two.

A. The Basic Solution for Pakistan

The development of the model described in Section II required a period of experimentation. It was necessary to determine a satisfactory form of the model in which a) the postulated objective function led to a rate of growth of national output similar to that taken as the objective of the Pakistan long-term plan, and b) implausible fluctuations in consumption and investment were eliminated. The end-product of these experiments is contained in relations (1) to (14) above. The
result of maximizing the welfare function subject to these fourteen constraints (for each time period) will be called the Basic Solution.

The initial conditions and structural parameters assumed in the Basic Solution are given in Tables II and III, which also present the corresponding values from the two versions of the Pakistan Perspective Plan where they are available. The welfare function parameter values used in the Basic Solution are given in Table IV.

The growth of national output in the Basic Solution is shown in Figure 1 and Table V to be approximately midway between the two versions of the perspective plan and therefore adequately representative of Pakistan objectives. The time paths of the other variables in the Basic Solution are shown in Figures 2 to 5. All values of variables and shadow price of the Basic Solution are presented in Tables VIIa and VIIb. Since the solution to the model does not distinguish between that part of trade-improving investment which is import substitution and that part which is export expansion, we have made an arbitrary distribution of trade improvement output for illustrative purposes.

The Basic Pattern of Investment and Capital Inflow

An examination of the binding constraints and their shadow prices shows that the twenty-three-year period of the Basic Solution can be divided into three sub-periods or "regimes". Each regime may be identified by the set of constraints which is binding. Since some are binding throughout (the limits on capacity, savings, and trade) the regimes can be described in terms of those which change.

This gives the following combinations in the Basic Solution.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Description</th>
<th>Distinguishing constraint</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Maximum investment and growth</td>
<td>Upper bound on rate of growth of investment (11)</td>
<td>1963-75</td>
</tr>
<tr>
<td>II</td>
<td>Trade improvement</td>
<td>Lower Bound on Rate of growth of investment (12)</td>
<td>1977-81</td>
</tr>
<tr>
<td>III</td>
<td>Balanced growth</td>
<td>No foreign capital (14)</td>
<td>1982-85</td>
</tr>
</tbody>
</table>

In the first regime the economy grows at the maximum rate permitted by the absorptive capacity limit on total investment, with only a small fraction allotted to import substitution. Since investment rises more rapidly than domestic

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14 We started from the preliminary version of the Pakistan Five-Year Plan for 1965-70 [6], and twenty-year perspective (Version 1 in Table II) and made some revisions after the final plan (Version 2) became available [7].

15 Trade-improvement production was allocated to export expansion so long as the rate of growth of exports did not exceed 6 per cent which was the export forecast in Version 1 [6]. When the 6 per cent level was reached, the remainder was allocated to import substitution.

16 This regime corresponds to phase I of Chenery and Strout [4].
Figure 1. Growth of GNP in Pakistan Perspective Plan and in two Solutions to Model
Figure 2. Savings and Investment in the Basic Solution
Figure 3. Exports and Imports in the Basic Solution
Figure 4. Composition of Investment in the Basic Solution
Figure 5. Optimal Patterns of Aid: Solutions 1 to 4 and 7
savings, the capital inflow increases steadily in this regime. The limit on external assistance—whether defined by its total or by the period over which it is available—causes the economy to shift to regime II in 1977. In this regime total investment ceases to grow and trade-improving investment \(^{(1)}\) rises to the proportion of the total needed to close the trade gap by the terminal year. As a result, the rate of growth of GNP slows down from its maximum of 8 per cent in 1975 to the rate which can be sustained by domestic savings in 1982 of about 6.3 per cent.

Regime III starts in the year in which aid is required to end. It is characterized by a proportion between trade-improving and total investment of about 1:4, which is just sufficient to prevent imports from outrunning exports. We have arbitrarily attributed enough of this investment to increased exports to achieve the steady growth of 6 per cent assumed in the first Pakistan Plan: the rest reduces import needs.

While the sharp transitions from one regime to another result from the use of a two-sector model with linear restrictions, this pattern of rising and then falling aid is a logical consequence of the high value of early increases in investment, income and savings for future growth. If the restriction on the rate of increase in investment were not imposed, the peaking of aid in the early years would be even more pronounced.

So long as the requirement that aid be terminated by the twentieth year (1982) is maintained, the Basic Solution is highly insensitive to variation in the relative valuation of plan and post-plan period consumption. With a price of foreign capital \(\gamma\) of 2 and the value of plan-period consumption as unity, the Basic Solution is the same for all values of the weight on terminal-year income \(\eta\) greater than one. Even when consumption during the plan is given no weight at all, the Basic Solution is not altered as long as the weight on terminal-year income is greater than 1.2.

The composition of investment over time is a consequence of the high productivity of aid in the early years. In order to absorb it, a gap has to be opened between imports and exports and then closed as rapidly as the rise in savings permits. Since the economy may not be sufficiently flexible to carry out this rapid structural change, observed growth paths—as shown in [4]—are likely to reflect a slower decrease in aid and a longer period of transition than the optimizing solution would suggest.

The Marginal Value of Aid. The Basic Solution can also be described as the solution to either of the other two forms of the model suggested above. Having determined the optimal amount of external capital corresponding to a twenty-year terminal period, we can take this amount as given in the third form of the
model. In the latter case, we do not specify a unit cost of the capital inflow, but determine its value as a result of the solution.

When the optimal solution is recalculated on these assumptions, the quantity solution is the same in all respects as that previously determined. The price solution differs in that the value of an additional unit of external capital is determined to be 7.4 instead of the 2.0 assumed initially. With this opportunity cost, the given amount of aid is distributed over time as shown in Figure 5 and reaches zero in twenty years without this condition being required in the model\textsuperscript{17}. The reasons for this pattern have been given above. In the Basic Solution with a specified terminal date (and undervalued external capital) the economy utilizes the maximum amount of aid that is consistent with the absorptive capacity constraint in regime I and the composition of investment required by regime II. The same solution will result for any preassigned unit value of foreign capital less than its true opportunity cost of 7.4.

This analysis of the Basic Solution shows that it could also be produced from the original form of the model if we assume initially a value of \( \gamma \) of 7.4. If Pakistan were offered unlimited amounts of capital at this (discounted) cost, its optimal development strategy would be to utilize this capital only over the next twenty years under the conditions specified in the model\textsuperscript{18}.

B. Variations in the Supply of External Capital

At the present time there is no coherent policy governing the total supply of external capital to underdeveloped countries. The procedures followed by the multilateral and bilateral lending institutions contain elements of three different allocation principles: \( i \) offers of loans at specified rates; \( ii \) rationing of assistance among countries on subjective criteria of need, performance, and political importance; \( iii \) planning of aid against a given terminal date. We now impose restrictions on the model to show the effect of alternative supply conditions on the optimal growth pattern and the social welfare.

Variation in Total Supply: We first determine the effects of varying the total capital inflow, assuming that its intertemporal distribution is unrestricted. A systematic variation in total supply can be specified with any of the three forms of the model by either \( i \) varying the parameter \( \gamma \) in the original welfare function; \( ii \) varying the total discounted amount of aid supplied; or \( iii \) varying the terminal date with no limitation on price or quantity. Our analysis of the Basic Solution shows that the set of solutions will be equivalent whichever approach is followed.

\textsuperscript{17} The cost of imposing this restriction would therefore be zero, whereas it was 11.2 in the first formulation. Otherwise the shadow prices in the two formulations are the same.

\textsuperscript{18} This statement is unrealistic in assuming constant supply and performance conditions over time.
The results of varying total discounted aid are shown in Figure 6a for values of \( F \) ranging from 10 per cent to 150 per cent of the amount in the basic solution. The corresponding variation in the value of aid (\( \gamma \)), is from 9.7 to 6.3 and in the terminal year from 4 years to 23 years.

The decline in the marginal productivity or value of aid results from the fact that as its quantity is increased, the use of external resources for investment purposes has to be postponed because of the limitation on absorptive capacity. This postponement reduces the amount of additional consumption and saving achieved per unit of additional aid during the plan period. On the other hand, since the value of future aid is discounted at 8 per cent, a dollar of aid today is equal in present value to \$4.5\) twenty years from now. For this reason, there is no decline in the marginal productivity of total discounted aid as measured by its effect on the terminal-year income although there would be a decline with a lower discount rate.

The two components of the welfare function are given separately in Table VI and Figure 6b to show these two effects. For any aid total, the marginal product in Figure 6a is equal to the sum of the marginal effects of aid on total consumption and terminal income with \( V_T \) given its appropriate weight.

The development sequence represented by the three regimes of the basic solution is unaffected by changes in the total amount of aid. As the total is reduced, the length of each of the first two regimes is shortened as indicated in Table VI. The effect on the optimal time path of aid is shown in Figure 5. Solution A-14 shows the effect of reducing the total aid by 50 per cent from the basic solution, and consequently shortening the period of aid from 20 to 14 years.

It is significant for assistance strategy that the optimal paths of all the variables are unaffected in regime I by an earlier termination date. Therefore a change in the total aid anticipated need not affect planning during this period.

Annual Rationing of Aid: The procedures by which public capital is currently supplied to developing countries result in a system of rationing in which there tends to be an absolute ceiling on the amount of aid furnished to any country in any one year. This ceiling can be represented in our model by limiting the annual inflow to a predetermined fraction of GNP. We will analyse the effects of such a limit in Pakistan by assuming a maximum of 5 per cent of GNP, which is approximately the average capital inflow in the past several years.

Solution B-20 in Table VI and Figure 5 show the effect of imposing this limitation in addition to the requirement of aid termination in twenty years.
Figure 6a. Marginal Productivity of Aid
Figure 6b. Composition of Total Welfare
The growth rate of the economy is reduced by 10 per cent and total capital inflow by about a third. The loss in welfare is significantly greater than would be the case if the same amount of aid were optimally distributed.

The effect of annual rationing with a given growth target is shown by solution B-30, which determines the amount of aid needed to achieve the same growth target as the Basic Solution with aid limited to 5 per cent of GNP. The result is to prolong the date of aid termination to 1992\(^{19}\), to increase total aid and to reduce total consumption as shown in Table VI. It is only for discount rates of greater than 9 per cent that there is any gain to the aid donors from this form of rationing. On an undiscounted basis the total aid required to achieve the given growth target is 65 per cent greater than with the optimum pattern\(^{20}\).

C. Variation in Development Performance

The most significant measures of a country’s development policies in the present model are the marginal rate of savings (\(\alpha\)), the absorptive capacity for investment (\(\beta\)) and the efficiency of use of capital (1/k). Since their effect on growth in closed models is well known, we are primarily interested in how they affect the productivity of aid and the amounts needed to obtain a given objective.

Figure 7 shows the variation in the marginal productivity of aid under the assumptions of a marginal savings rate of 0.16 instead of 0.24 (curve C) and of an absorptive capacity of 0.20 instead of 0.13 (curve D)\(^{21}\). Other assumptions of the Basic Solution are unchanged. These curves can be compared to the productivity of the Basic Solution (curve A). The time path of aid for a terminal year of 20 is similarly compared to the Basic Solution in Figure 8\(^{21}\).

Higher absorptive capacity raises the marginal productivity by an increasing amount as the total aid is increased. At the level of the Basic Solution, the same growth target could be achieved with about 10 per cent less aid. Alternatively, a growth rate of 8.3 per cent could be achieved for the plan period compared to the 6.6 per cent of the Basic Solution. Even with the doubling of aid that this increase would require, its marginal productivity would remain higher than in the Basic Solution.

A fall in the marginal savings rate from 0.24 to 0.16 would lower the marginal productivity most substantially at low levels of aid. For a given terminal year (solution C-20) the lower savings performance reduces the terminal-year income by about 25 per cent with only a small reduction in total discounted aid. It is clear that the ability of a country to save and reinvest a substantial proportion of

\(^{19}\) Solution A-20 shows the Basic Solution projected to 1992.

\(^{20}\) In the optimal pattern, aid reaches a peak of 10.5 per cent of GNP in the fourteenth year.

\(^{21}\) Table VI gives other characteristics of the solutions.
Figure 7. Marginal Productivity of Aid
SOLUTION A-20 BASIC SOLUTION (SEE TABLE 2)
SOLUTION C-20 MARGINAL SAVINGS RATE (κ) = .16
OTHERWISE SAME AS NO. 1.
SOLUTION D-20 MAXIMUM RATE OF GROWTH OF INVESTMENT
(ρ) IS 20% OTHERWISE SAME AS NO. 1.

Figure 8. Optimal Patterns of Aid Solutions 1, 6, and 7
its increases in income is one of the most important reasons why external assistance can be highly productive\textsuperscript{22}.

Finally, we can make a brief comment on the most significant difference between our analysis and the Pakistan Perspective Plan. As shown in Figure 1, our solution B-20 (with aid limited to 5 per cent of GNP) closely parallels the growth of GNP in the more conservative preliminary version of the perspective plan\textsuperscript{23}. This solution requires about 50 per cent more external capital than the preliminary plan mainly because of our assumption that trade improvement will require substantially more capital than is indicated by the marginal coefficient of 3.0 that has been experienced recently. Since the problems of closing the balance of payments gap are not explicitly analysed in the perspective plan, we cannot explore this difference further.

\section*{IV. DEVELOPMENT AND ASSISTANCE STRATEGY}

Although our experimentation with this model has not proceeded far enough to test the generality of our results, there are several aspects of development strategy which seem to apply under a wide variety of assumptions. The first is the high productivity of early increases in investment and consequently of the external resources which make them possible. Our optimum investment patterns bear a striking resemblance to those of Goodwin [5], even though the savings rate necessary to sustain them is held within realistic limits by the availability of external resources. The main function of aid is thus to permit an economy to grow at a rate determined by its ability to invest rather than by its initial ability to increase savings.

For aid donors interested in achieving either self-sustaining growth or a given growth target, the assistance provided will be considerably more effective if it permits the recipient to follow this optimum strategy of rapid growth in the early years, which permits a shorter period of assistance for any given target.

There is a strong indication that the optimal growth strategy while investment is rising in regime I is not dependent on the total aid to be provided. In our example, Pakistan's optimum policy until 1969 would be the same either with the aid expected in the Basic Solution or with half that much. This suggests the possibility of conditional planning by donors and recipients in which the aid of subsequent years could depend on initial performance without distorting investment decisions in the earlier years. The implications of this conclusion need to be tested in more realistic models.

\footnotetext{\textsuperscript{22} Estimates of the productivity of assistance over shorter periods are given in [1;3;4].} \textsuperscript{23} See, Figure 1.
Finally, the possibility of measuring the productivity of assistance from an analysis of a country's development possibilities suggests a line of improvement in the procedure for intercountry allocation of aid. The marginal productivity curves of Figures 6 and 7 can be interpreted as demand curves for external capital, which could be helpful in rationing any given amount of foreign assistance. The use of such measures would focus attention on the aspects of both donor and recipient performance that are most important to successful development.

REFERENCES


TABLE I

VARIABLE AND PARAMETER DEFINITION

Variables:

\( V \)  =  gross national product

\( V^1 \)  =  production for import substitution and export expansion

\( V^o \)  =  all other production

\( I \)  =  total gross investment

\( I^1 \)  =  investment in import substitution and export expansion

\( I^o \)  =  all other investment

\( S \)  =  savings

\( F \)  =  net capital inflow

\( M \)  =  demand for traditional imports*

\( E \)  =  traditional exports*

\( C \)  =  consumption

Parameters:

\( \gamma \)  =  cost of foreign capital exogenously specified

\( i \)  =  rate of discount

\( \rho \)  =  post-plan growth rate

\( r \)  =  rate of discount on post-plan consumption

\( s \)  =  weight on post-plan consumption

\( \eta \)  =  weight for terminal year income incorporating discount procedure for future consumption

\( e \)  =  exogenous rate of growth of traditional exports

\( k_1 \)  =  capital-output ratio for import substitution and export expansion

\( k_o \)  =  capital-output ratio for other production

\( \alpha \)  =  marginal savings rate

\( m_o \)  =  marginal import rate on income

\( m_1 \)  =  marginal import rate on investment

\( \beta \)  =  maximum feasible rate of growth of investment

\( p \)  =  minimum allowable rate of growth of consumption

\( T \)  =  terminal year of the plan

\( T-n \)  =  year in which aid must cease

---

* Traditional imports and traditional exports mean imports which would be required and exports which could be sold were the structure of the economy to remain unchanged from the base year.
TABLE II

BASE YEAR DATA

(\textit{in million rupees, 1965})

<table>
<thead>
<tr>
<th></th>
<th>Model values</th>
<th>Pakistan plan values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_0$</td>
<td>Foreign Aid</td>
<td>1183 \hspace{1cm}</td>
</tr>
<tr>
<td>$S_0$</td>
<td>Savings</td>
<td>3381 \hspace{1cm}</td>
</tr>
<tr>
<td>$I_0$</td>
<td>Investment</td>
<td>4564 \hspace{1cm}</td>
</tr>
<tr>
<td>$M_0$</td>
<td>Imports</td>
<td>3743 \hspace{1cm}</td>
</tr>
<tr>
<td>$E_0$</td>
<td>Exports</td>
<td>2559 \hspace{1cm}</td>
</tr>
<tr>
<td>$V_0$</td>
<td>National Income</td>
<td>37380 \hspace{1cm}</td>
</tr>
<tr>
<td>$C_0$</td>
<td>Consumption</td>
<td>33999 \hspace{1cm}</td>
</tr>
</tbody>
</table>

\textit{Sources:} Version 1 from [6].
Version 2 from [7].

\textit{Note:} Model values are averages derived from a time trend for the years 1957-1962, which were thought to be more representative than the actual data for 1962.
### TABLE III

**VALUE OF STRUCTURAL PARAMETERS**

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ Marginal savings rate</td>
<td>0.24</td>
<td>0.286</td>
<td>0.25</td>
<td>0.23</td>
<td>0.22</td>
<td>0.26</td>
<td>0.25</td>
<td>0.30</td>
<td>0.28</td>
<td>0.31</td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_o$ Marginal import rate on income</td>
<td>0.10</td>
<td>0.072</td>
<td>0.06</td>
<td>na</td>
<td>0.12</td>
<td>na</td>
<td>0.09</td>
<td>na</td>
<td>0.06</td>
<td>na</td>
<td>0.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_1$ Marginal import rate on investment</td>
<td>0.35</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_o$ Incremental capital-output ratio, regular production</td>
<td>3.0</td>
<td>3.6</td>
<td>2.9</td>
<td>3.5</td>
<td>2.9</td>
<td>3.5</td>
<td>2.9</td>
<td>3.6</td>
<td>2.9</td>
<td>3.7</td>
<td>3.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_1$ Incremental capital-output ratio, trade improvement</td>
<td>4.5</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P$ Rate of population growth</td>
<td>2.5</td>
<td>**</td>
<td>2.6</td>
<td>2.6</td>
<td>2.7</td>
<td>2.7</td>
<td>2.8</td>
<td>2.6</td>
<td>2.6</td>
<td>2.2</td>
<td>2.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$ Maximum rate of growth of Investment</td>
<td>0.13</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td>na</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e$ Rate of growth of exports (per cent)</td>
<td>4.9</td>
<td>6.0</td>
<td>7.9</td>
<td>6.0</td>
<td>9.5</td>
<td>6.0</td>
<td>8.7</td>
<td>6.0</td>
<td>8.6</td>
<td>6.0</td>
<td>4.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Sources:* Version 1 from [6]  
Version 2 from [7]

*Note:* Model—$m_o$, $m_1$, and $k_o$ were estimated from time trends for 1957-62; $e$ and $\alpha$ were modified to reflect improved performance in 1963 and 1964.


**TABLE IV**

NON-STRUCTURAL PARAMETERS IN THE BASIC SOLUTION

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>0.08 rate of discount during plan period</td>
</tr>
<tr>
<td>$r$</td>
<td>0.10 rate of discount on post-plan consumption</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.073 post-plan rate of growth</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2 cost of foreign capital</td>
</tr>
<tr>
<td>$\eta$</td>
<td>3.4 defined in the text</td>
</tr>
<tr>
<td>$\delta$</td>
<td>1 relative valuation of post-plan consumption</td>
</tr>
<tr>
<td>$T$</td>
<td>23 terminal year of plan</td>
</tr>
<tr>
<td>$T-n$</td>
<td>20 year in which aid must cease</td>
</tr>
</tbody>
</table>

**TABLE V**

GROWTH RATES AND SIGNIFICANT RATIOS FOR THE BASIC SOLUTION OF THE MODEL AND THE TWO VERSIONS OF THE PAKISTAN PLAN

<table>
<thead>
<tr>
<th>Years</th>
<th>Plan</th>
<th>Rate of growth (per cent)</th>
<th>$V_n/V_o$</th>
<th>$I_n/V_n$</th>
<th>$S_n/V_n$</th>
<th>$F_n/V_n$</th>
<th>$I_n^{1}/I_n$</th>
</tr>
</thead>
</table>

**BASIC SOLUTION**

<table>
<thead>
<tr>
<th>Years</th>
<th>Plan</th>
<th>Rate of growth (per cent)</th>
<th>$V_n/V_o$</th>
<th>$I_n/V_n$</th>
<th>$S_n/V_n$</th>
<th>$F_n/V_n$</th>
<th>$I_n^{1}/I_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1965-70</td>
<td>III</td>
<td>5.9</td>
<td>1.33</td>
<td>0.21</td>
<td>0.14</td>
<td>0.07</td>
<td>0.05</td>
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<tr>
<td>1970-75</td>
<td>IV</td>
<td>7.7</td>
<td>1.45</td>
<td>0.27</td>
<td>0.17</td>
<td>0.10</td>
<td>0.07</td>
</tr>
<tr>
<td>1975-80</td>
<td>V</td>
<td>8.0</td>
<td>1.47</td>
<td>0.23</td>
<td>0.19</td>
<td>0.03</td>
<td>0.39</td>
</tr>
<tr>
<td>1980-85</td>
<td>VI</td>
<td>6.3</td>
<td>1.36</td>
<td>0.21</td>
<td>0.21</td>
<td>0.00</td>
<td>0.21</td>
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</table>

**PAKISTAN PLAN—VERSION 1**

<table>
<thead>
<tr>
<th>Years</th>
<th>Plan</th>
<th>Rate of growth (per cent)</th>
<th>$V_n/V_o$</th>
<th>$I_n/V_n$</th>
<th>$S_n/V_n$</th>
<th>$F_n/V_n$</th>
<th>$I_n^{1}/I_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1965-70</td>
<td>III</td>
<td>5.4</td>
<td>1.30</td>
<td>0.19</td>
<td>0.13</td>
<td>0.06</td>
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<tr>
<td>1970-75</td>
<td>IV</td>
<td>5.9</td>
<td>1.33</td>
<td>0.20</td>
<td>0.16</td>
<td>0.04</td>
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<tr>
<td>1975-80</td>
<td>V</td>
<td>6.7</td>
<td>1.38</td>
<td>0.22</td>
<td>0.20</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>1980-85</td>
<td>VI</td>
<td>6.8</td>
<td>1.39</td>
<td>0.24</td>
<td>0.23</td>
<td>0.01</td>
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**PAKISTAN PLAN—VERSION 2**

<table>
<thead>
<tr>
<th>Years</th>
<th>Plan</th>
<th>Rate of growth (per cent)</th>
<th>$V_n/V_o$</th>
<th>$I_n/V_n$</th>
<th>$S_n/V_n$</th>
<th>$F_n/V_n$</th>
<th>$I_n^{1}/I_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1965-70</td>
<td>III</td>
<td>6.7</td>
<td>1.38</td>
<td>0.20</td>
<td>0.14</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>1970-75</td>
<td>IV</td>
<td>7.3</td>
<td>1.43</td>
<td>0.21</td>
<td>0.17</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>1975-80</td>
<td>V</td>
<td>7.5</td>
<td>1.44</td>
<td>0.22</td>
<td>0.20</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>1980-85</td>
<td>VI</td>
<td>7.5</td>
<td>1.44</td>
<td>0.23</td>
<td>0.22</td>
<td>0.01</td>
<td></td>
</tr>
</tbody>
</table>

* Source of Pakistan Plan data same as for Table II. In this table the subscript n refers to the final year of the particular plan and the subscript o refers to the first year of the particular plan.
### TABLE VI

**EFFECTS OF VARYING AID SUPPLY**

(in billion rupees)

<table>
<thead>
<tr>
<th>Solution</th>
<th>Aid measures</th>
<th>Benefit measures</th>
<th>Characteristics of development patterns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total aid</td>
<td>Shadow price</td>
<td>Total aid un-</td>
</tr>
<tr>
<td></td>
<td>discounted 9%</td>
<td>of discounted</td>
<td>discounted</td>
</tr>
<tr>
<td>1. Variation of total aid supply</td>
<td>9.5</td>
<td>9.3</td>
<td>12.9</td>
</tr>
<tr>
<td>A—12</td>
<td>19.8</td>
<td>8.4</td>
<td>33.4</td>
</tr>
<tr>
<td>A—16</td>
<td>38.9</td>
<td>7.4</td>
<td>90.5</td>
</tr>
<tr>
<td>A—20</td>
<td>57.4</td>
<td>6.7</td>
<td>172.1</td>
</tr>
<tr>
<td>A—24</td>
<td>38.9</td>
<td>—</td>
<td>90.5</td>
</tr>
</tbody>
</table>

2. Variation of total supply with annual aid limited to 5% of GNP

<table>
<thead>
<tr>
<th>Solution</th>
<th>Aid measures</th>
<th>Benefit measures</th>
<th>Characteristics of development patterns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total aid 20%</td>
<td>Shadow price</td>
<td>Total aid un-</td>
</tr>
</tbody>
</table>

3. Variation of total supply and variation of performance

<table>
<thead>
<tr>
<th>Solution</th>
<th>Aid measures</th>
<th>Benefit measures</th>
<th>Characteristics of development patterns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total aid</td>
<td>Shadow price</td>
<td>Total aid un-</td>
</tr>
<tr>
<td>C—13</td>
<td>12.9</td>
<td>7.3</td>
<td>19.8</td>
</tr>
<tr>
<td>C—20</td>
<td>30.6</td>
<td>7.0</td>
<td>60.6</td>
</tr>
<tr>
<td>D—11</td>
<td>16.5</td>
<td>9.1</td>
<td>23.9</td>
</tr>
<tr>
<td>D—20</td>
<td>83.0</td>
<td>7.6</td>
<td>19.6</td>
</tr>
</tbody>
</table>

**Key to Table**

The solutions to the model are designated by a letter and a number. The letter indicates the form of the model and the number indicates the year in which self-sustaining growth begins.

- A—solutions using the parameters and form of the basic model.
- B—solutions in which the annual capital inflow is limited to 5 per cent of GNP.
- C—solutions in which the marginal savings rate (α) is 0.16. Otherwise the same as A.
- D—solutions in which the limit on the rate of growth of investment (β) is 0.20. Otherwise the same as A.

Example: A—20 is the Basic Solution.
<table>
<thead>
<tr>
<th>Plan Year</th>
<th>Net capital inflow</th>
<th>Gross national product</th>
<th>Regular production</th>
<th>Trade improvement production</th>
<th>Total gross investment</th>
<th>Regular investment</th>
<th>Trade improvement investment</th>
<th>Savings</th>
<th>Consumption</th>
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* Trade improvement production was allocated to export expansion so long as the rate of growth of exports did not exceed 6 per cent which was the export forecast in Version 1. When the 6 per cent level was reached, the remainder was allocated to import substitution.
### TABLE VIIb

**SHADOW PRICES IN THE BASIC SOLUTION**

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