‘ALLAIS PARADOX AND ITS PARADOXICAL IMPLICATION

In real life, some people pay to assume risk while some others pay to get rid of risk. Furthermore, each person is neither exclusively risk-averse nor risk-seeking; rather he/she chooses some risky and some safe acts at the same time. The same person who puts his life at risk climbing mountains may refuse to drive a car without his seat belt on or to invest in stocks, because he considers them to be too risky. Therefore, it is extremely difficult to determine empirically the dominant behavior.

Examples of Risky and Safe Acts

Risky Acts

*Purchase of lottery tickets

*Betting on games

*Gambling

*Purchase of stocks & junk bonds

*preference for self employment & for job in a private company

Safe Acts

*Purchase of life, car & fire insurance policy

*Keeping money in bank deposits & govt. bonds

*preference for govt. job

Diminishing Marginal Utility of Wealth, Risk Aversion & Choice under Uncertainty

- The disutility derived from a dollar's loss is always greater than the utility derived from a dollar's gain. It means risk aversion or the cumulative utility-of-wealth function of every person is concave.
- Therefore, to compare competing options, maximize expected utility rather than expected payoffs. $EU = \Sigma U(x_i)\, p_i$ where x_i denotes possible payoffs $x_1, x_2, \ldots, x_n$; $U(x_i)$ denotes utility derived from payoff x_i and p_i denotes probability of occurrence of x_i such that $\Sigma p_i = 1$
- The option, which has the highest value of EU, is ranked at the top and so on.

Expected Utility Hypothesis (EUH) and Mean-Variance (m-v) Criterion

- EUH is though knotted with risk aversion that is $U''(x_i) < 0$, yet it can also be used to reflect neutral and risk-seeking attitudes through increasing straight line or $U''(x_i) = 0$ and through increasing convex curve or $U''(x_i) > 0$.
- As an analytical tool, EUH is preferred over m-v because m-v involves comparison of two parameters of each option and it does not work if both mean and variance of one option are greater than those of the other.

Allais Paradox

Option A		Option A*		Option B		Option B*	
Payoff (million$)	Probability	Payoff (million$)	Probability	Payoff (million$)	Probability	Payoff (million$)	Probability
1	1.00	0	0.01	0	0.89	0	0.90
		1	0.89	1	0.11	5	0.10
		5	0.10				
E(A) = 1 var(A) = 0		E(A*) = 1.39 var(A*) = 1.46		E(B) = 0.11 var(B) = 0.10		E(B*) = 0.50 var(B*) = 2.25	

- Out of AA*, majority chose A . It means U(1) ≥ .01 U(0) + .89 U(1) + .10 U(5) or .11 U(1) ≥ .01 U(0) + .10 U(5) ….. (1)
- Out of BB*, majority chose B* . It means .89 U(0) + .11 U(1) ≤ .90 U(0) + .10 U(5) or 0.11 U(1) ≤ 0.01 U(0) + 0.10 U(5) ….. (2)
- Expressions on both sides of (1) and (2) are same but the inequality sign is reversed, therefore it shows a contradiction of EUH or 'Allais paradox.'

Why is ‘Allais Paradox’ a paradox for EUH?

- It not only contradicts EUH with $U''(x_i) < 0$ but it also contradicts EUH with $U''(x_i) = 0$ and $U''(x_i) > 0$.
- Moreover, risk aversion of either of the two pairs cannot be tested by m-v criterion. In m-v, option $A^*$is preferred over option A if $E(A) \leq E(A^*)$ and $var(A) \geq var(A^*)$ whereas in this case $E(A) < E(A^*)$ and $var\ (A) < var\ (A^*)$.
- Therefore, it should not have been posed as a paradox for EUH with $U''(x_i) < 0$.

Paradoxical Nature of ‘Allais Paradox’

- It contradicts EUH but it further strengthens the conviction of risk aversion intuitively. Option A out of AA^* is chosen because of the certainty effect that promotes risk aversion.
- Many authors reaffirmed ‘Allais paradox’ but they did not check EUH and risk aversion simultaneously. They came up with more exceptions to EUH such as the certainty effect, big amount effect, common consequence effect, common ratio effect, reverse common ratio effect, response mode effect, framing effect and hypothetical vs. cash mode effect.
- In other words, Allais and subsequent researches have created, on one side, doubts about EUH but, on the other side, they have supported risk aversion intuitively as the norm.
- As a result, EUH remains the main criterion for decision making under uncertainty in most of the textbooks, though the number of exceptions to EUH rule is also increasing.

Possible Reason for Confusion

- The argument of diminishing marginal utility of wealth holds if the distribution is symmetric, otherwise it may not hold because the certainty and the big amount effects, which counter each other, are not the same for every person.

Illustration of Certainty and Big Amount Effects by Hypothetical Bets

Bet A			Bet B		
Payoffs	Probability	E(P)	Payoffs	Probability	E(P)
-10	0.45	-4.50	-10	0.80	-8.00
0	0.10	0	0	0.10	0
10	0.45	4.50	80	0.10	8.00

- in bet A, the amount to lose and to win is same and their probability is also same. Hence, the argument holds.
- In bet B, in left, the certainty effect is strong as p = .8 but in right tail, the big amount effect is strong (80 > 10). Therefore, a person who cares more about certainty will not participate in this bet. However, another person who cares more about a big amount will participate in this bet.

Recommendation

- Further research should concentrate more on risk aversion than on EUH. It can better be achieved by posing such option-pairs which are useful to test EUH and risk aversion simultaneously as I did.
- If both EUH and risk aversion are violated, then economic modeling should inculcate risk averse as well as risk-seeking attitude.

Simultaneous Testing of 'Allais Paradox' and Risk Aversion

	A		A^*		B		B^*		Choice Pattern (%)			
	Payoff	Prob.	Payoff	Prob.	Payoff	Prob.	Payoff	Prob.	AB	AB^*	A^*B	A^*B^*
1	-10 20 50 (56%)	0.167 0.667 0.167	-10 20 50	0.333 0.333 0.333	-10 20 (62%)	0.333 0.667	-10 50	0.667 0.333	42	14	20	24
2	-10 0 20 (56%)	0..167 0.500 0.333	-10 0 50	0.333 0.500 0.167	-10 20 100 (57%)	0..167 0.333 0500	-10 50 100	0.333 0.167 0.500	43	13	14	32
3	0 10 15 (51%)	0.1 0. 7 0.2	0 10 15	0.2 0.4 0.4	0 10 (55%)	0.4 0.6	0 15	0.6 0.4	31	20	24	25
4	0 5 (56%)	0.5 0.5	0 10	0.75 0.25	0 5 (63%)	0.6 0.4	0 10	0.8 0.2	43	13	20	24

Experimental Results

- In the 4 choice-sets, a majority of respondents ranging from 51% to 56% preferred option A over option A^* and a majority ranging from 55% to 63% preferred option B over option B^*. It means that a majority of respondents is risk averse.
- However, a risk-averse attitude is not confirmed from the choice-pattern of respondents that emerges from their combined choice for both option-pairs in each choice-set as given in the last 4 columns of table. Though the highest percentage of respondents, ranging from 31% to 43%, showed risk-averse behavior consistently in both options of each choice-set, this percentage significantly falls short of the 50% threshold.
- An important implication is that risk aversion should not be treated as the exclusive behavior; it can be used at best as a good working assumption for choice under uncertainty. Economists should rather attempt to develop decision-making criteria which takes into account mixed and risk-loving behaviors as well