Fully Modified HP Filter

By

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Outline

- Introduction
- Issues with HP filter
- MHP filter
- End Point Bias
- Existing Solution (extrapolation)
- Critical evaluation of extrapolating time series
- Innovative Solution - Fully Modified HP Filter
- Simulation Study
- Empirical Results
- Concluding Remarks

Introduction [to smoothing a time series]

- Smoothing a time series is very common amongst Economists for studying business cycles
- Simplest approach to estimate trend is moving average
- Sophisticated approaches are like HP filter.

Issues with HP Filter and Solutions

- Fixed Value of Lambda. This issued is addressed by McDermott (1997) and evaluated by Choudhary et al (2014) – Modified HP Filter (endogenous lambda).
- End Point Bias (EPB). This issue needs to be solved:
  1. Solution suggested in the literature is to *extrapolate the given time series* and then apply the filtering technique. We assessed this approach in one paper by conducting a simulation as well as empirical study.
  2. Better solution is to directly hit the bias and minimize it. We introduce a Fully Modified HP filter in this study (endogenous lambda and endogenous weighting scheme).

Understanding EPB in HP/MHP Filter

- HP Filter [Hodrick Prescott (1997)] to MHP Filter [McDermott (1997)]

$$y_t = g_t + c_t, \quad t = 1, 2, 3, \ldots, T \qquad (1)$$

HP filter estimates a cyclical series (c_t) by minimizing the sum of square of difference between series (y_t) and its trend part (g_t) subject to the constraint that the squared sum of dynamic differences of the trend is not too large.

$$min[\sum_{t=1}^{T}(y_t - g_t)^2 + \lambda \sum_{t=2}^{T-1}[(g_{t+1} - g_t) - (g_t - g_{t-1})]^2] \qquad (2)$$

$$\hat{g}_t = [I + \lambda A]^{-1} y_t \qquad (3)$$

Where $A = K'K$ where $K = \{k_{ij}\}$ is a (T-2) ×T matrix with elements as given below

$$k_{ij} = \begin{cases} 1 & \text{if } j = i \text{ or } j = i + 2, \\ -2 & \text{if } j = i + 1, \\ 0 & \text{otherwise} \end{cases}$$

Modified HP filter of McDermott (1997) relaxes this (assumption of) fixed values of λ as explained in Choudhary et.al (2014). The idea of this procedure is to apply HP filter method (in equation 3) by excluding a single data point at a time and select a λ which gives best fit of the data point left out. The emphasis therefore is on selecting an optimal value of λ with reference to the subject time series.

The optimal value of λ can be obtained by minimizing the following equation with respect to λ.

$$GCV(\lambda) = T^{-1}(1 + \frac{2T}{\lambda}) \sum_{k=1}^{T}(y_k - g_{t,k}(\lambda))^2 \qquad (4)$$

In this way smoothing parameter stands endogenous (to observed data).

End Point Bias

In order to carry out a trend-cycle decomposition of a time series at a given date, HP filtering requires information about the behavior of the series at earlier as well as at later dates. Absence of end and start observations (as is evident from equation 2) poses difficulties at the start and the end of the sample resulting changes in terminal points weights and thus causing in substantial distortions in cyclical component at both ends. This is what has been termed as end-point bias in the literature.

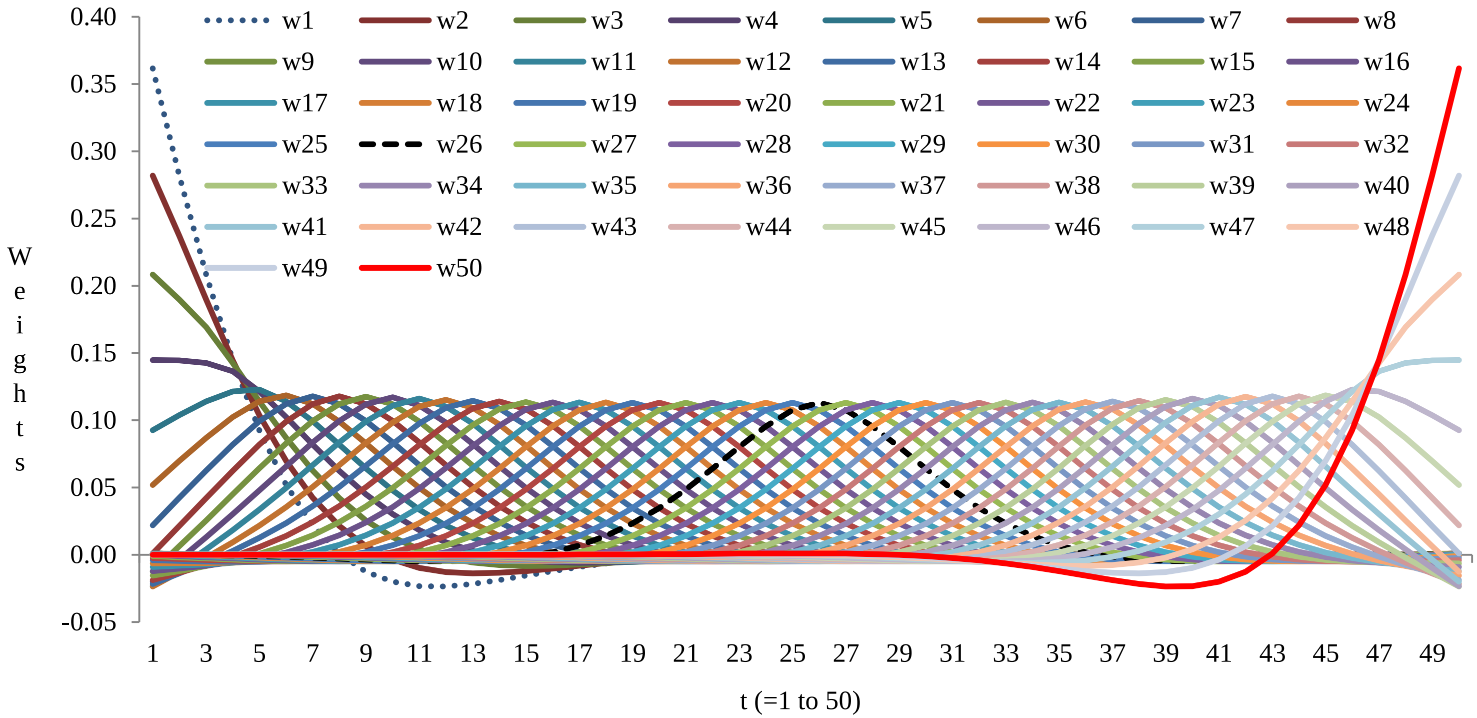
HP filter is a symmetric filter in the sense that the estimator $\hat{g}_t$ (equation 3) is the weighted sum of both lags and leads of y_t. Due to the missing values at both ends, the whole weighted matrix B in equation (3) is distorted with highest effect on boundaries and lowest at the middle of data set. This can be seen from Figure (1) where we plot the ‘weighting vectors’ corresponding to $\hat{g}_t$ for a HP-filter with $\lambda = 100$ applied to a series with 50 data points.

End Point Bias (Lambda = 100, T=50)

0.36	0.28	0.21	0.14	0.09	0.05	0.02	0.00	-0.01	-0.02	-0.02	-0.02	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.28	0.24	0.19	0.14	0.10	0.07	0.04	0.02	0.01	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.21	0.19	0.17	0.14	0.11	0.09	0.06	0.04	0.03	0.01	0.00	-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.14	0.14	0.14	0.14	0.12	0.10	0.08	0.06	0.04	0.03	0.02	0.00	-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.09	0.10	0.11	0.12	0.12	0.11	0.10	0.08	0.06	0.05	0.03	0.00	0.00	0.00	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.05	0.07	0.09	0.10	0.11	0.12	0.11	0.10	0.08	0.07	0.05	0.01	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.02	0.04	0.06	0.08	0.10	0.11	0.12	0.11	0.10	0.08	0.07	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.02	0.04	0.06	0.08	0.10	0.11	0.12	0.11	0.10	0.08	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-0.01	0.01	0.03	0.04	0.06	0.08	0.10	0.11	0.12	0.11	0.10	0.04	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-0.02	0.00	0.01	0.03	0.05	0.07	0.08	0.10	0.11	0.12	0.11	0.05	0.04	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-0.02	-0.01	0.00	0.02	0.03	0.05	0.07	0.08	0.10	0.11	0.12	0.07	0.05	0.04	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.01	0.01	0.02	0.04	0.05	0.11	0.11	0.11	0.10	0.08	0.06	0.05	0.03	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.01	0.01	0.02	0.04	0.10	0.11	0.11	0.11	0.10	0.08	0.06	0.05	0.03	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-0.01	-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.01	0.01	0.02	0.08	0.10	0.11	0.11	0.11	0.10	0.08	0.06	0.05	0.03	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	-0.01	-0.01	0.00	0.00	0.00	0.00	0.01	0.01	0.06	0.08	0.10	0.11	0.11	0.11	0.10	0.08	0.06	0.05	0.03	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.01	0.05	0.06	0.08	0.10	0.11	0.11	0.11	0.10	0.08	0.06	0.05	0.03	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.05	0.06	0.08	0.10	0.11	0.11	0.11	0.10	0.08	0.06	0.05	0.03	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.03	0.05	0.06	0.08	0.10	0.11	0.11	0.11	0.10	0.08	0.06	0.05	0.03	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.05	0.06	0.08	0.10	0.11	0.11	0.11	0.10	0.08	0.06	0.05	0.03	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.05	0.06	0.08	0.10	0.11	0.11	0.11	0.10	0.08	0.06	0.05	0.03	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.05	0.06	0.08	0.10	0.11	0.11	0.11	0.10	0.08	0.06	0.05	0.03	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.05	0.06	0.08	0.10	0.11	0.11	0.11	0.10	0.08	0.06	0.05	0.03	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.05	0.06	0.08	0.10	0.11	0.11	0.11	0.10	0.08	0.06	0.05	0.03	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.05	0.06	0.08	0.10	0.11	0.11	0.11	0.10	0.08	0.06	0.05	0.03	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.05	0.06	0.08	0.10	0.11	0.11	0.11	0.10	0.08	0.06	0.05	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.05	0.06	0.08	0.10	0.11	0.11	0.11	0.10	0.08	0.06	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.05	0.06	0.08	0.10	0.11	0.11	0.11	0.10	0.08	0.06	0.01	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.05	0.06	0.08	0.10	0.11	0.11	0.11	0.10	0.08	0.01	0.01	0.00	0.00	0.00	0.00	-0.01	-0.01	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.05	0.06	0.08	0.10	0.11	0.11	0.11	0.10	0.02	0.01	0.01	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.05	0.06	0.08	0.10	0.11	0.11	0.11	0.04	0.02	0.01	0.01	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.05	0.06	0.08	0.10	0.11	0.11	0.05	0.04	0.02	0.01	0.01	0.00	0.00	-0.01	-0.01	-0.01	-0.01
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.04	0.05	0.12	0.11	0.10	0.08	0.07	0.05	0.03	0.02	0.00	-0.01	-0.02
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.04	0.11	0.12	0.11	0.10	0.08	0.07	0.05	0.03	0.01	0.00	-0.02
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.10	0.11	0.12	0.11	0.10	0.08	0.06	0.04	0.03	0.01	-0.01
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.08	0.10	0.11	0.12	0.11	0.10	0.08	0.06	0.04	0.02	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.07	0.08	0.10	0.11	0.12	0.11	0.10	0.08	0.06	0.04	0.02
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.05	0.07	0.08	0.10	0.11	0.12	0.11	0.10	0.09	0.07	0.05
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01	0.00	0.00	0.03	0.05	0.06	0.08	0.10	0.11	0.12	0.12	0.11	0.10	0.09
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	0.02	0.03	0.04	0.06	0.08	0.10	0.12	0.14	0.14	0.14	0.14
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.01	0.03	0.04	0.06	0.09	0.11	0.14	0.17	0.19	0.21
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	0.00	0.01	0.02	0.04	0.07	0.10	0.14	0.19	0.24	0.28
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.02	-0.02	-0.01	0.00	0.02	0.05	0.09	0.14	0.21	0.28	0.36

End Point Bias

Figure 1: Weighting vectors of HP filter with Lambda 100



We can see that as we go to the middle of data set we have symmetric weighting vector and to the boundaries the filter weights become more and more asymmetric. It can be observed that the highest weight at the margins is disproportionately large compared to those at the middle (of the time series). Hence the estimation at the end points is effected by disproportionately large weights at terminal points. This behavior of HP filter weighting scheme causes the biasness at (up to 20 observations on) both ends in the extracted cyclical component.

End Point Bias

To further understand the distortion at the terminal points of the estimated trend component of time series using HP filter, we can consider HP filter in frequency domain.

Going back to equation (3) we see filter weights are contained in matrix $[I + \lambda A]^{-1} = B_{T\times T}$, where B is symmetric matrix and its t^{th} row contains the weights for the estimation $\hat{g}_t$

$$\hat{g}_t = \sum_{j=1}^{T} b_{tj} y_j \qquad (5)$$

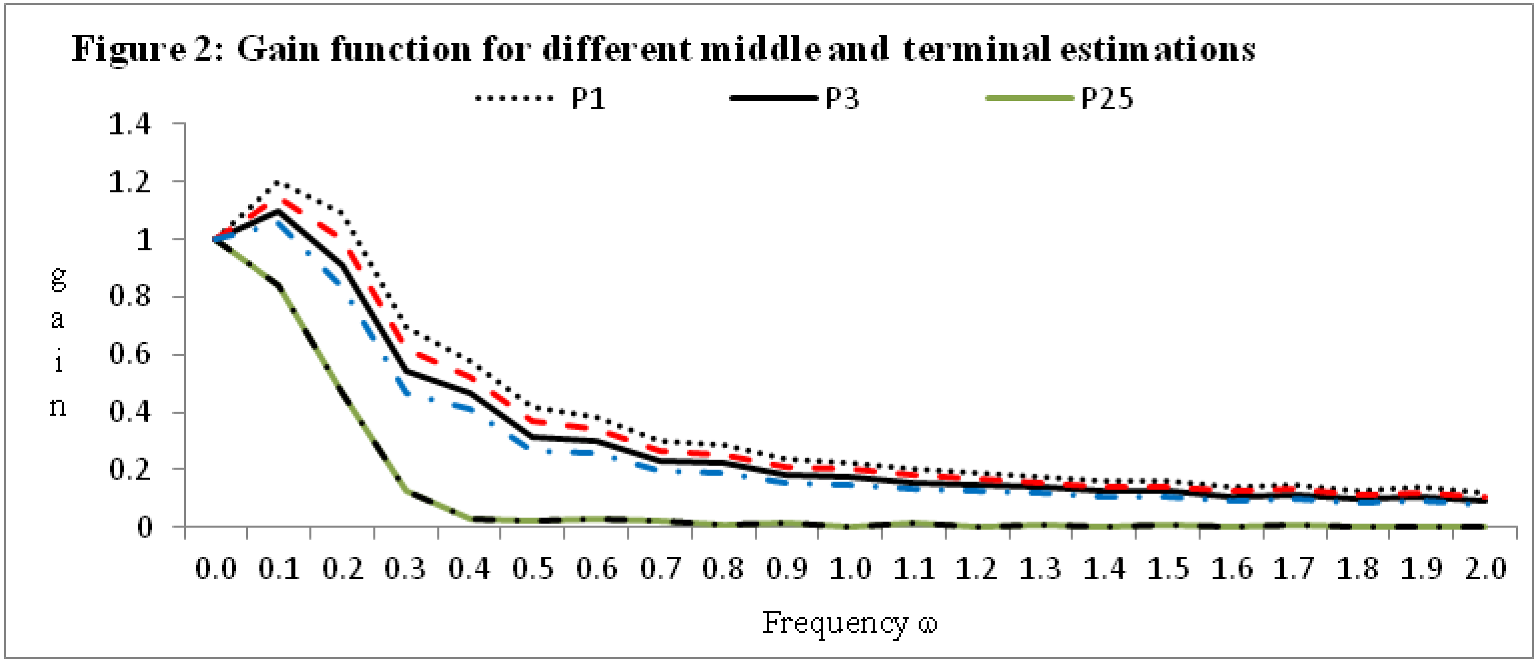
Where b_{tj} is the j^{th} element of the t^{th} row of B. These weights have symmetric structure in the middle and change its shape near the boundaries.

In the frequency domain a time series is interpreted as the overlap of oscillations with different frequencies where the trend as a long run behavior (of a series) is supposed to consist of those oscillations with high periodicity. In the estimation of trend component, HP filter extract oscillations with high periodicities and eliminates oscillations with lower periodicities. This behavior can be explained by a gain function. By using filter weights in matrix B for a given value of lambda, the gain for estimation $\hat{g}_t$ at different frequency level can be calculated as

$$P_t(\omega, \lambda) = \sqrt{\left(\sum_{j=1-t}^{T-t} b_{t,j+t} \cos(\omega j)\right)^2 + \left(\sum_{j=1-t}^{T-t} b_{t,j+t} \sin(\omega j)\right)^2} \qquad (6)$$

Bloechl (2014) explained that this gain can be interpreted as a factor by which the amplitude of an oscillation with a certain frequency is decreased or increased by a filter. Taking T=50 we plotted this gain (P_t) in Figure 2 for t=1, 3, 25, 26, 47, and 49. By construction (of HP filter) weights are different for middle and terminal points.

End Point Bias



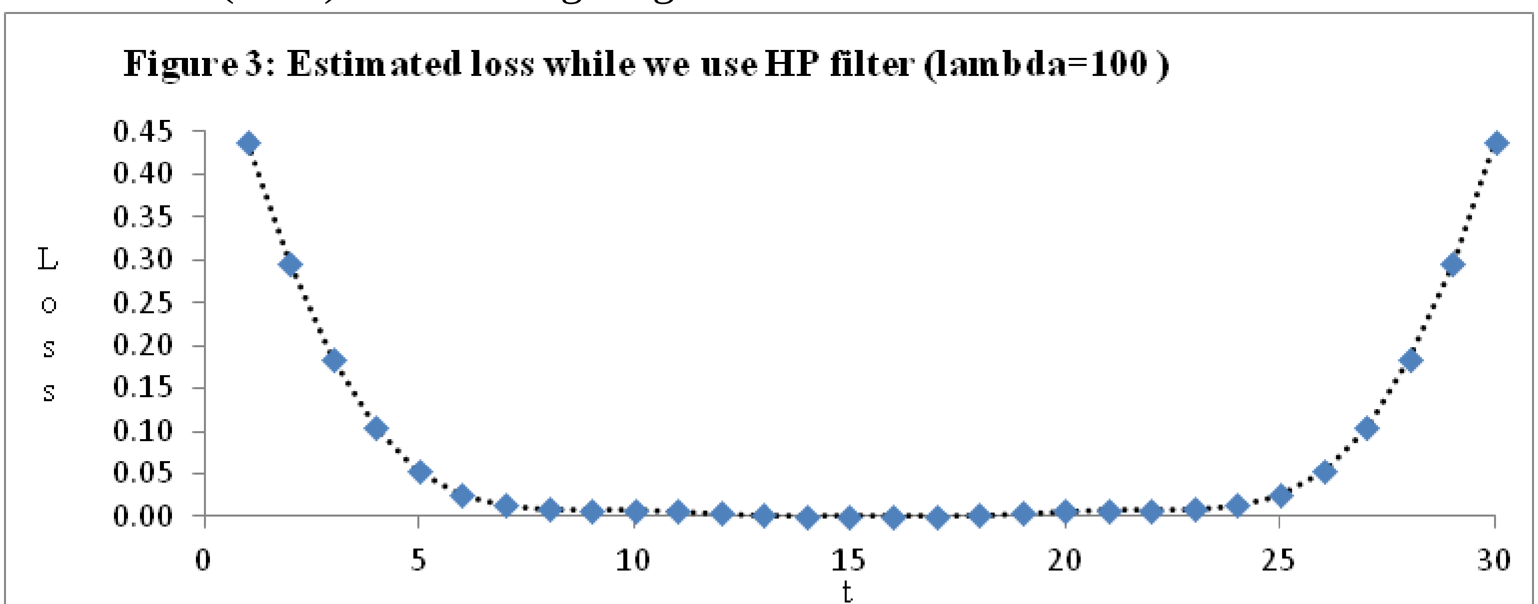
We can see in this figure how gain is affected by changing the weights (i.e. for middle and terminal points): for the estimations at the middle (like, 25th and 26th) we see very similar gain as they depend upon an almost equal weight structure and the gain starts to change as we move to the boundaries (like for the 1st, 3rd, 47th and 49th estimations). Thus, for the estimation at end points the high frequencies cannot be completely eliminated anymore which causes an increasing volatility in the trend component. So the trend estimates at terminal points contain part of the cyclical component and is thus distorted. Resulting in larger than should be standard deviation in trend component (and hence lower than should be standard deviation in cyclical component).

End Point Bias

To quantify the distortion at the terminal data points in trend component Bloechl (2014) introduced a loss measure in the form of deviation of gain function of certain estimation $\hat{g}_t$ from the one at the middle (centre) where we know distortion is negligible. This loss is what one would like to minimize. If $P_c(\omega, \lambda)$ denotes the gain for frequency ω and parameter λ for the centre estimation $\hat{g}_c$, where c=T/2, and $P_t(\omega, \lambda)$ is the gain for estimation $\hat{g}_t$ then the loss function is

$$l(t, \lambda) = \sum_{i=1}^{n} [P_c(\omega_i, \lambda) - P_t(\omega_i, \lambda)]^2 . \theta \qquad (7)$$

We use ω=(0, 0.1, 0.2, . . ., $\pi)^{/}$. Here, n is number of elements in ω and θ is the distance between the element in ω i.e. $\theta = \omega_j - \omega_{j-1} = 0.1$. Calculating the loss for all t=1, 2, . . .,T gives an overview of distortions at the estimations (for the trend) on terminal points (Figure 3). To eliminate the EPB, the ideal weighting scheme would be the one which gives zero overall loss. We discuss Bloechl (2014) and our weighting schemes to address the EPB later.



Existing Solution (extrapolation)

One way to handle this issue is to extend data from both ends (Mohr (2005)) before applying HP filter to decompose the time series of interest. There are different ways to extend data. Kaiser and Maravall (1999) and Denis et al (2002) suggested to use ARIMA (p,d,q) model for extrapolating the data at both ends.

Critical Evaluation of Existing Solution

We think this is not a proper solution of end point biasness in HP filtering as the choice of DGP for extending the subject series in itself is biased simply because we do not know the true DGP of the series of interest.

Any other solution?

Yes

Review of Past Attempts to hit EPB directly

Bruchez (2003) proposed a new mechanism of changing the weighting scheme in the HP filter: for certain values of t, g_t term appears less often in the second part of equation (2) so increase the corresponding value of λ for those values of t. More specifically, since first and last value appears only once, 2nd and 2nd last value appears twice and all other values appears three times in second part of equation (2), he proposed to multiply lambda by 3 for first and last values, and by 3/2 for second and second last values. We believe that Bruchez (2003) approach to handle EPB also has shortcomings including a) Bruchez (2003) uses arbitrary numbers (3 and 3/2) to change the weights for the terminal points, and b) he completely ignored the weighting issues in other than the terminal four values.

Review of Past Attempts to hit EPB directly

Bloechl (2014) introduced a new weighting scheme for the end values of the data but not arbitrarily like Bruchez (2003). He introduced a loss function to minimize (as discussed above). In order to resolve end points asymmetrical weighting issue of HP filter, Bloechl (2014) suggested (i) flexible scheme for number of end observations (k) to consider and (ii) flexible weights for end observations (α).

With the loss function (Equation 7) one can assess the distortion which causes the biasness at the terminal points' estimates (of trend using HP filter). Bloechl (2014) developed a scheme to reduce this distortion: higher the loss, higher the penalization (in linear manner). Thus, Bloechl (2014) suggested a flexible penalization for HP filter by taking different values of λ for different points in time. Considered a cumulative loss function:

$$L(\lambda) = \sum_{t=1}^{T} l(t, \lambda) \tag{8}$$

Review of Past Attempts to hit EPB directly

Bloechl (2014) implemented his scheme by replacing the scalar λ in equation (3) with a vector λ_t while increasing λ for k values at both ends. That is

$$\lambda_{T-2-k+j} = \lambda_{HP} + \alpha j, \quad \text{and } \lambda_{k-j+1} = \lambda_{HP} + \alpha j, \qquad j = 1, \ldots, k \tag{9}$$

With different choices of α and k for new vector λ_t in equation (9), one can obtain different values of accumulated loss function. Based upon his simulation work Bloechl (2014) has given different choices of α and k for different values of λ and for different time period. Bloechl (2014) found estimated loss with his suggested scheme (1.16872) to be lower than that for HP filter approach (1.76382).

Review of Past Attempts to hit EPB directly

There are various issues in the way Bloechl (2014) attempted to address the EBP. First of all it considers only linear penalization to minimize the loss function in equation 8 whereas figures 1 to 3 clearly suggest possibility of nonlinear penalization to minimize the loss function to zero. Moreover, selecting seemingly optimal weight (α) from amongst the arbitrarily (and thus exogenously) chosen values of this weight does not ensure the loss minimization.

Fully Modified HP Filter

We marry the endogenous lambda approach of McDermott (1997) with loss function minimization approach of Bloechl (2014) while suggesting some intuitive changes in his weighting scheme (we critically evaluated above). We contribute by suggesting an endogenous weighting scheme along with endogenous smoothing parameter and resolve EPB issue of HP filter. We call this *fully modified* (FMHP) *filter*.

We first estimate the lambda endogenously. For this, we estimate $g_{t,k}(\lambda)$ (i.e., equation 3) by applying the leave-out method (of McDermott, 1997) with λ=1 as an initial value. For different positive values of λ, we estimate equation (4) and select λ that gives the minimum value of the objective function in equation 4. We call this λ^{MHP}. By this time we have an endogenous smoothing parameter. Here we propose the following (improved) scheme to minimize the cumulative loss in equation 8: (i) use linear or non linear increase of penalization (whichever minimizes the cumulative loss in equation 8) to the terminal points, (ii) fix the value of k (=20) and (iii) endogenous weights (for end observations) i.e. endogenous α.

Fully Modified HP Filter (steps)

- First, estimate $g_{t,k}(\lambda)$ applying the leave-out method using equation 3 starting with an arbitrary value for λ (=1);
- Second, for different values for λ>0, we obtain different estimates of (4) and λ that gives the minimum value of the objective function (4) is chosen as the initial smoothing parameter as in McDermott (1997), λ^{MHP}.
- Third, using value of λ^{MHP} and weighting scheme below, we obtain optimal value of α and k.

$$\lambda_{T-2-k+j}^{MHP} = \lambda^{MHP} + \alpha j^{i}, \text{ and } \lambda_{k-j+1}^{MHP} = \lambda^{MHP} + \alpha j^{i}, \quad j = 1, \dots, k; \ i=1, 2. \qquad (9a)$$

- Fourth, repeat step first to re-estimate $g_{t,k}(\lambda)$ with new weighting scheme using equation 3 and optimal value of α and k from step 3;
- Fifth and last, using $g_{t,k}(\lambda)$ from step 4 along with different values of λ and new weighting scheme below we obtain different values of $g_{t,k}(\lambda)$ and hence different values of equation (4). And λ that gives the minimum value of the objective function (4) is chosen as the optimal smoothing parameter, λ^{FMHP}.

$$\lambda_{T-2-k+j}^{FMHP} = \lambda^{FMHP} + \alpha j^{i}, \text{ and } \lambda_{k-j+1}^{FMHP} = \lambda^{FMHP} + \alpha j^{i}, \quad j = 1, \dots, k; \ i=1, 2. \qquad (9b)$$

Has FMHP Filter solved EPB?

Figure 1: Weighting vectors of HP filter with Lambda 100

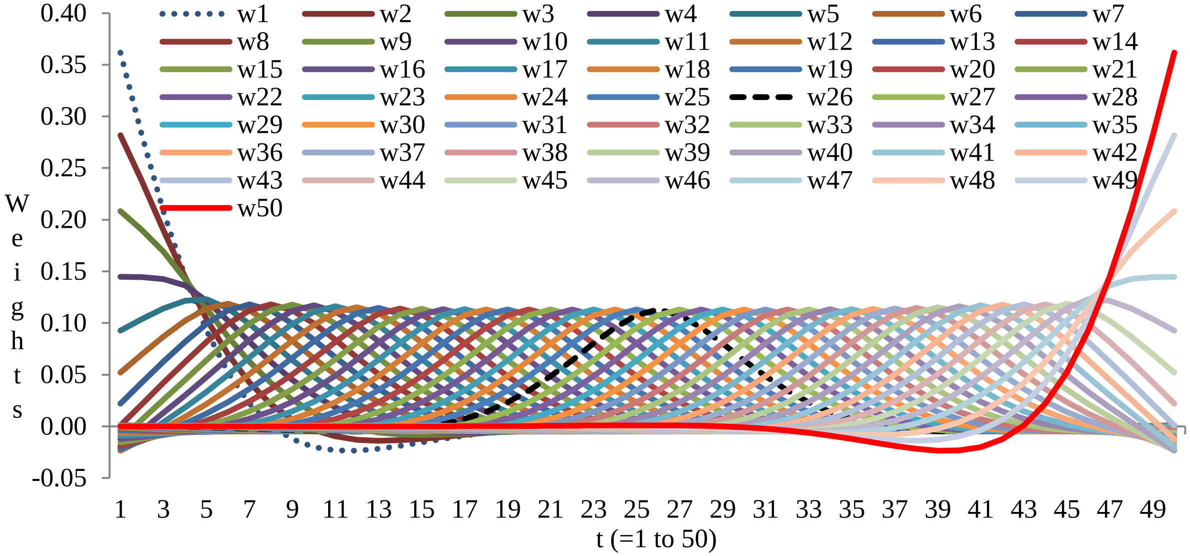
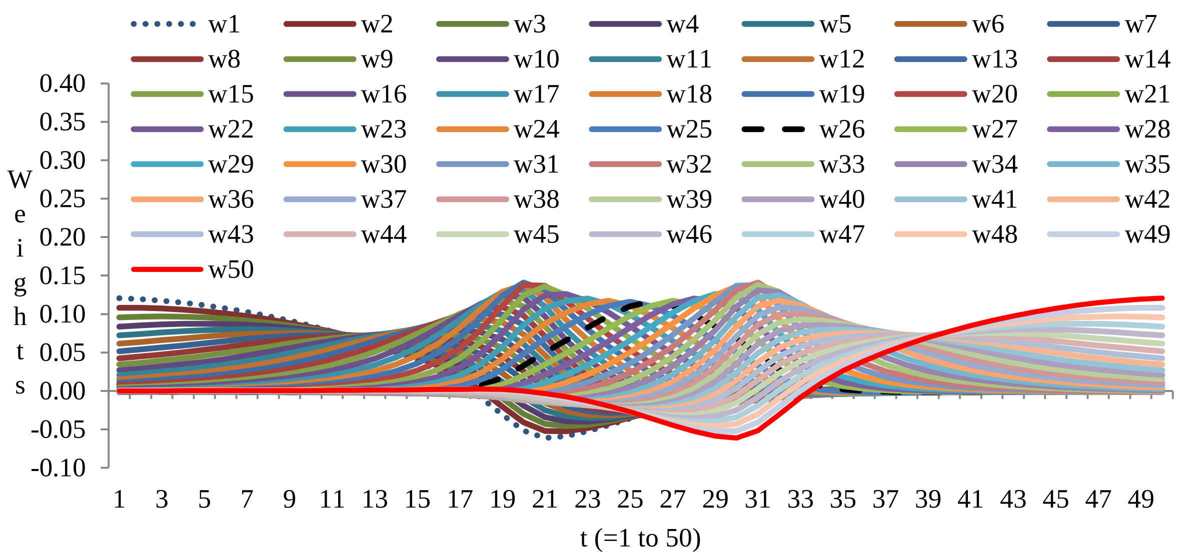


Figure 1a: Weighting vectors of our scheme with lambda 100



Has FMHP Filter solved EPB?

Figure 2: Gain function for terminal and middle observations (using HP filter with lambda=100)

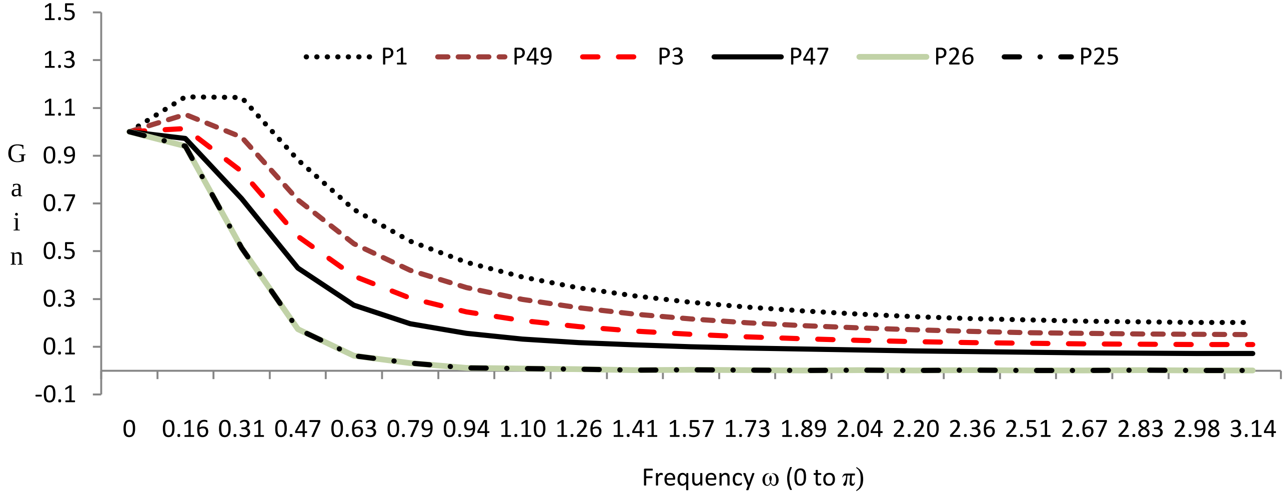
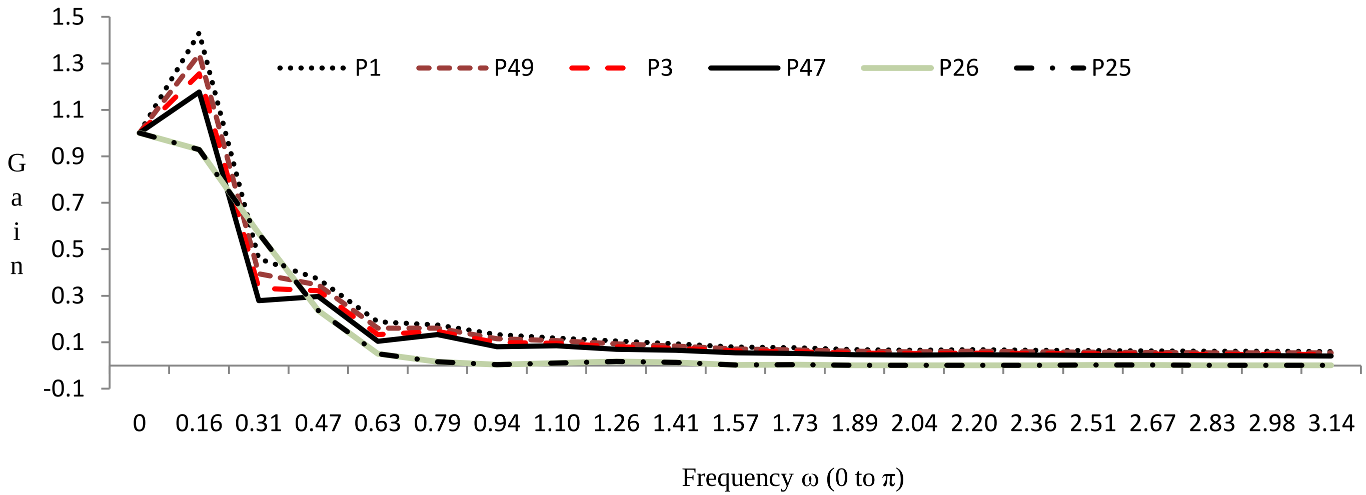


Figure 2a: Gain function for terminal and middle observations using our weighting scheme (with α=10 and k=20) with lambda=100



Has FMHP Filter solved EPB?

Figure 3: Estimated loss while we use HP filter (lambda=100)

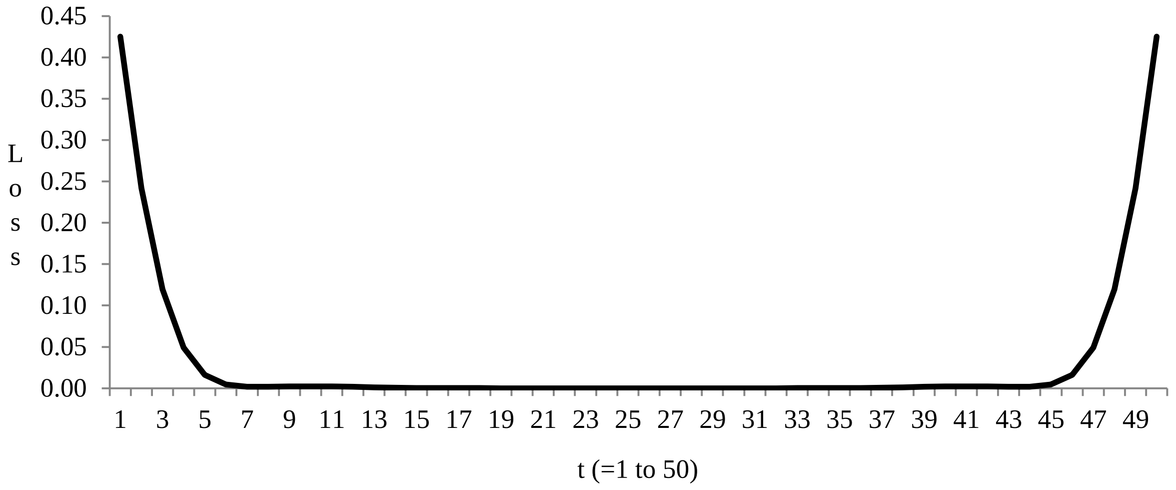
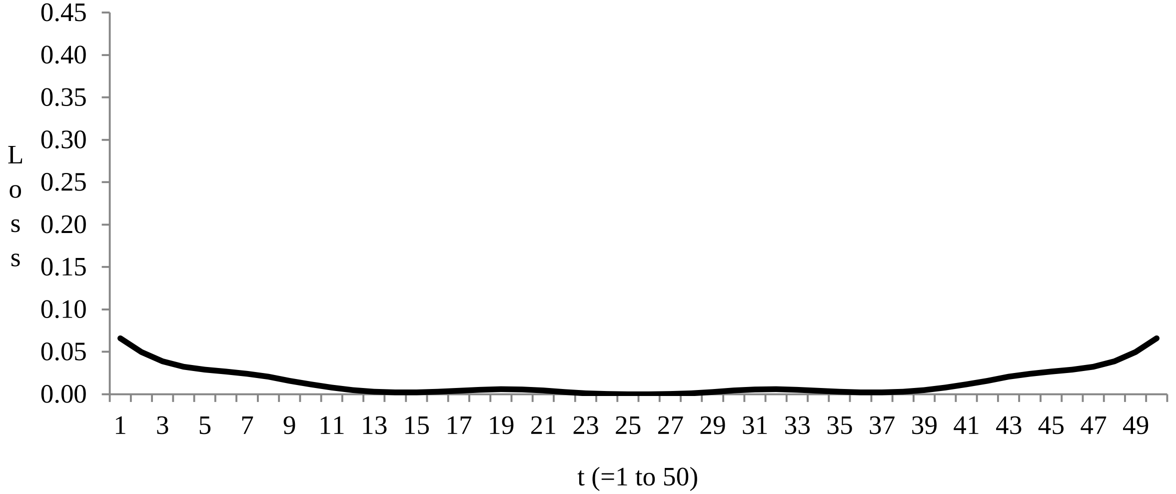


Figure 3a:Estimated loss using our weighting scheme (α=10, k=20) with lambda=100



Simulation Design

Our simulation exercise has two stages: (i) generation of artificial series, and (ii) use of artificially generated series to evaluate HP, BK, CF and this study's FMHP filters. Following Hodrick and Prescott (1997), we know that a typical economic time series (x_t) is composed of a trend (g_t) and a cycle (c_t) i.e. $x_t = g_t + c_t, \quad t = 1, 2, 3, \ldots, T$. By choosing suitable data generating process (DGP), as discussed below, trend and cyclical components are generated separately. These two components are then combined to obtain single time series. This single time series is later decomposed using each of the above listed filters (i.e. HP, BK, CF, and FMHP). We compare the performance of these filters in extracting the cyclical part of the series. We use the root mean squared error (RMSE) as performance criterion. Ideally it should be zero. We actually see the abilities of these four filters to estimate cyclical components at end points of data series as well as on the middle in order to assess which filter performs the best particularly in minimizing the EPB.

Simulation Design

The trend and cyclical components for quarterly data can be generated as

$$g_t = \text{drift} + \text{trend}_t + g_{t-1} + \varepsilon_t \quad (10a)$$

$$c_t = \theta_1 c_{t-1} + \theta_2 c_{t-2} + \delta_t \quad (10b)$$

Where $\varepsilon_t \sim NIID(0, \sigma_\varepsilon^2)$, $\delta_t \sim NIID(0, \sigma_\delta^2)$.

The data-generating process of equations 10a and 10b is chosen on the evidence that the trend of most observed macroeconomic series tends to follow a random walk with a drift, which could be either linear or nonlinear, while the cyclical series follows an AR(2) process. The DGP has general specification where trend part satisfies the unit root condition while cyclical part follows the stationary process [with $\theta_1 + \theta_2 < 1$ and $|\theta_2| < 1$].

We also consider the change of relative importance of each component by varying the ratio of standard deviation,$\sigma_\varepsilon/\sigma_\delta$, of the disturbances in equations 10a and 10b. In order to generate the artificial data closer to some observations (we have) upon real life data we take these ratios slightly different from Choudhary et.al (2014) and Guay and St. Amant (2005). We consider the following values of the ratio $\sigma_\varepsilon/\sigma_\delta$: 10, 5, 2, 1 and 0.50.

Simulation Results

Table 1: Simulation Results of Performance[1] Comparison of fully modified HP filter with HP, BK, and CF fitters[2]

Mode	$(\sigma_\varepsilon / \sigma_\xi)$	AR Coefficients		Percent of times fully modified HP filter outperforms HP, (BK[6]), [CF[7]] filter							
				(Generated as) Quarterly		Time Aggregated (Annual)					
						Systematically		By Summing		By Averaging	
		First ($\emptyset_1$)	Second ($\emptyset_2$)	Linear trend	Non linear trend	Linear trend	Non linear trend	Linear trend	Non linear trend	Linear trend	Non linear trend
(a)	b[3]	c[3]	d[3]	e[4]	f[4]	g[5]	h[5]	i[5]	j[5]	k[5]	l[5]
1	10	0.9	0.01	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
2	10	1.2	-0.25	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
3	10	1.2	-0.4	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
4	10	1.2	-0.55	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
5	10	1.2	-0.75	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
6	5	0.9	0.01	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
7	5	1.2	-0.25	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
8	5	1.2	-0.4	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
9	5	1.2	-0.55	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
10	5	1.2	-0.75	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
11	2	0.9	0.01	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
12	2	1.2	-0.25	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
13	2	1.2	-0.4	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
14	2	1.2	-0.55	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
15	2	1.2	-0.75	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
16	1	0.9	0.01	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
17	1	1.2	-0.25	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
18	1	1.2	-0.4	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
19	1	1.2	-0.55	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
20	1	1.2	-0.75	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
21	0.5	0.9	0.01	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
22	0.5	1.2	-0.25	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
23	0.5	1.2	-0.4	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
24	0.5	1.2	-0.55	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
25	0.5	1.2	-0.75	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
26	10	0.8	0	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
27	5	0.8	0	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
28	2	0.8	0	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
29	1	0.8	0	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]
30	0.5	0.8	0	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]	100, (100), [100]

1: Performance criterion is the root mean square error of the artificial cyclical and estimations of those artificial cyclical series. 2: HP, BK and CF denote Hodrick-Prescott, Baxter-King and Christiano-Fitzgerald filters. 3. Columns b-d presents model's assumptions for generating the artificial data. 4. Columns e-f are the power of fully modified HP filter compared to HP, BK and CF filters for quarterly data generated by linear and non-linear models respectively. 5. Columns g to l represent the power of fully modified HP filter compared to HP, BK and CF filters for time aggregated (by summing as well as averaging) annual data generated by linear and non-linear models. 6. For BK filter, we assume fixed lead-lag length, k=12 for quarterly data and k=3 for annual data; maximum length of cycle P1 =32 for quarterly and P1=8 for annual data; minimum length of cycle P2=6 for quarterly data and P2=2 for annual data. 7. For CF filter there is no need for a fixed lag length: maximum and minimum length of cycles is the same as that of the BK filter.

None of HP, BK and CF filter could beat our FMHP filter even in a single model single time in this power comparison study.

Simulation Results

Table 2: Root Mean Square Error of Cyclical component estimated by Fully Modified HP, HP, BK and CF filter

		Average RMSE of cyclical component of 30 models											
		Generated data set (full)				Generated data set (middle values 80%)				Generated data set (end points 20%)			
		FMHP	HP	BK	CF	**FMHP**	HP	BK	CF	**FMHP**	HP	BK	CF
(Generated as) Quarterly		**3.0**	13.0	39.9	57.1	**2.3**	3.8	19.9	17.3	**5.6**	17.1	NA	73.9
Time Aggregated (Annual)	Systematically	**8.7**	33.8	18.1	33.7	**3.6**	17.0	17.8	10.9	**17.4**	67.4	NA	72.2
	By Summing	**23.3**	135.1	72.6	135.9	**10.1**	68.4	71.3	43.5	**46.3**	269.4	NA	291.2
	By Averaging	**5.9**	33.8	18.2	34.0	**2.5**	17.1	17.9	10.9	**11.7**	67.3	NA	72.8

We can see that RMSE for end points is significantly high than the RMSE at the middle of the data set for all the filtering techniques used here. Hence for both quarterly and annually data series, all these filters have upward bias at end points of the series - for example, in case of quarterly data set the RMSE of HP filter at 'terminal points' is 350% high than the RMSE at the middle. For FMHP this increase is 160% which is less than half (as compared to HP filter's 350%). Hence FMHP filter has smallest bias while HP and CF filters have higher EPB for quarterly and annual data respectively. We also observe from Table that FMHP has 'overall' lowest RMSE as compared to other methods for quarterly as well as annual data sets. *Hence EPB is reduced significantly by using FMHP filter*.

Simulation Results

Table 2a: Root Mean Square Error of Cyclical component estimated by Fully Modified & Wavelet Analysis with extrapolation (WAN WE)

		Average RMSE of cyclical component of 30 models					
		Generated data set (full)		Generated data set (middle values)		Generated data set (end points)	
		FMHP	WAN (WE)	**FMHP**	WAN (WE)	**FMHP**	WAN (WE)
(Generated as) Quarterly		**3.0**	8.3	**2.3**	8.9	**5.6**	78.2
Time Aggregated (Annual)	Systematically	**8.7**	24.3	**3.6**	23.9	**17.4**	92
	By Summing	**23.3**	57.0	**10.1**	56.3	**46.3**	337
	By Averaging	**5.9**	14.5	**2.5**	14.4	**11.7**	76

Empirical Design

We use annual and quarterly time series of three core macroeconomic variables namely real GDP, real (private) consumption and real investment. We select those 70 countries for which at least 40 annual observations for each of these series is available. Quarterly national income accounts being scant, we could find quarterly time series for income, consumption and investment for 33 countries only. The quarterly data is seasonally adjusted. We grouped all the countries into four income categories: high, upper middle, lower middle and lower income (as per World Bank 2015 classification). All the series are transformed into logarithms before we proceed to decompose the observed time series.

Empirical Results

Table 3a: Estimated Loss (Annual Data)

	Fixed Lambda (100)			This study proposed (our) weighing scheme with endogenous Lambda		
	HP Filter	HP (Bloechl scheme)	HP (our scheme)	FMHP filter	FMHP filter	FMHP filter
	Income	Income	Income	Income	Consumption	Investment
Algeria	1.735	1.260	0.626	0.545	0.532	0.560
Australia	1.735	1.260	0.626	0.523	0.560	0.539
Austria	1.733	1.267	0.545	0.513	0.491	0.494
Bangladesh	1.735	1.260	0.626	0.521	0.535	0.637
Belgium	1.733	1.267	0.545	0.506	0.507	0.468
Benin	1.735	1.260	0.626	0.537	0.525	0.540
Bolivia	1.735	1.260	0.626	0.586	0.567	0.533
Botswana	1.731	1.147	0.647	0.505	0.501	0.505
Brazil	1.735	1.260	0.626	0.561	0.565	0.536
Burkina Faso	1.732	1.272	0.623	0.508	0.482	0.484
Cameroon	1.735	1.260	0.626	0.576	0.534	0.606
Canada	1.733	1.267	0.545	0.520	0.501	0.476
Chile	1.735	1.260	0.626	0.556	0.529	0.522
Colombia	1.735	1.260	0.626	0.522	0.529	0.580
Congo, Rep	1.735	1.260	0.626	0.536	0.602	0.539
Costa Rica	1.735	1.260	0.626	0.546	0.546	0.552
Cuba	1.733	1.256	0.534	0.482	0.482	0.507
Cyprus	1.731	1.147	0.647	0.471	0.391	0.391
Denmark	1.733	1.267	0.545	0.492	0.522	0.470
Dominican Republic	1.735	1.260	0.626	0.541	0.527	0.542
Ecuador	1.735	1.260	0.626	0.561	0.541	0.537
Egypt	1.733	1.225	0.604	0.470	0.423	0.397
El Salvador	1.732	1.272	0.623	0.513	0.506	0.503
Finland	1.733	1.267	0.545	0.517	0.481	0.540
France	1.733	1.267	0.545	0.505	0.506	0.489
Gabon	1.735	1.260	0.626	0.522	0.529	0.522
Germany	1.733	1.267	0.545	0.477	0.472	0.504
Greece	1.733	1.267	0.545	0.550	0.531	0.480
Guatemala	1.735	1.260	0.626	0.584	0.582	0.526
Honduras	1.735	1.260	0.626	0.523	0.521	0.560
Hong Kong	1.733	1.227	0.546	0.484	0.481	0.403
India	1.735	1.260	0.626	0.526	0.534	0.524
Indonesia	1.735	1.260	0.626	0.550	0.530	0.522
Iran	1.735	1.260	0.626	0.565	0.593	0.566
Ireland	1.733	1.267	0.545	0.480	0.488	0.512
Italy	1.733	1.267	0.545	0.477	0.476	0.474
Japan	1.733	1.256	0.534	0.487	0.483	0.481
Kenya	1.733	1.262	0.627	0.503	0.606	0.494
Lesotho	1.733	1.262	0.627	0.512	0.532	0.498
Luxembourg	1.733	1.267	0.545	0.476	0.510	0.509
Madagascar	1.735	1.260	0.626	0.521	0.546	0.548
Malaysia	1.735	1.260	0.626	0.529	0.620	0.560
Malta	1.733	1.225	0.604	0.474	0.375	0.366
Mauritania	1.735	1.260	0.626	0.528	0.521	0.560
Mexico	1.735	1.260	0.626	0.562	0.526	0.523
Morocco	1.734	1.278	0.623	0.522	0.510	0.541
Netherlands	1.733	1.267	0.545	0.481	0.486	0.505
New Zealand	1.733	1.267	0.545	0.484	0.479	0.472
Norway	1.735	1.260	0.626	0.568	0.526	0.551
P.N. Guinea	1.733	1.256	0.534	0.489	0.487	0.457
Peru	1.735	1.260	0.626	0.571	0.561	0.590
Philippines	1.735	1.260	0.626	0.583	0.572	0.542
Portugal	1.733	1.267	0.545	0.504	0.486	0.472
Puerto Rico	1.733	1.227	0.546	0.471	0.471	0.475
Rwanda	1.735	1.260	0.626	0.531	0.531	0.539
Senegal	1.735	1.260	0.626	0.522	0.527	0.563
Singapore	1.731	1.147	0.647	0.470	0.417	0.469
South Africa	1.735	1.260	0.626	0.582	0.569	0.566
South Korea	1.735	1.260	0.626	0.559	0.544	0.531
Spain	1.733	1.267	0.545	0.536	0.545	0.494
Sudan	1.735	1.260	0.626	0.525	0.521	0.557
Sweden	1.733	1.267	0.545	0.509	0.487	0.473
Thailand	1.735	1.260	0.626	0.552	0.521	0.529
Togo	1.735	1.260	0.626	0.524	0.678	0.523
Trinidad & Tobago	1.735	1.260	0.626	0.581	0.575	0.595
Tunisia	1.734	1.278	0.623	0.508	0.497	0.502
UK	1.733	1.267	0.545	0.516	0.492	0.510
Uruguay	1.732	1.272	0.623	0.484	0.481	0.528
USA	1.733	1.256	0.534	0.495	0.482	0.445
Venezuela	1.733	1.262	0.627	0.500	0.497	0.496
Average	**1.734**	**1.256**	**0.599**	**0.522**	**0.518**	**0.514**

Empirical Results

Table 3b: Estimated Loss (Quarterly Data)

	Fixed Lambda (100)			This study proposed (our) weighing scheme with endogenous Lambda		
	HP Filter	HP (Bloechl scheme)	HP (our scheme)	FMHP filter	FMHP filter	FMHP filter
	Income	Income	Income	Income	Consumption	Investment
Australia	1.762	1.618	0.818	0.829	0.881	0.815
Austria	1.761	1.259	0.786	0.616	0.741	0.806
Belgium	1.766	1.495	0.797	0.649	0.686	0.616
Brazil	1.761	1.259	0.786	0.798	0.698	0.800
Canada	1.760	1.608	0.818	0.659	0.669	0.683
Costa Rica	1.769	1.682	0.814	0.657	0.720	0.911
Czech Rep	1.761	1.259	0.786	0.582	0.641	0.652
Denmark	1.766	1.495	0.797	0.547	0.608	0.571
Estonia	1.766	1.495	0.797	0.589	0.594	0.642
Finland	1.769	1.682	0.818	0.710	0.729	0.684
France	1.762	1.617	0.820	0.610	0.668	0.573
Germany	1.769	1.682	0.814	0.747	0.798	0.637
Greece	1.766	1.495	0.797	0.685	0.694	0.696
Hungary	1.766	1.495	0.797	0.623	0.644	0.739
India	1.757	1.125	0.770	0.663	0.750	0.651
Ireland	1.757	1.125	0.770	0.622	0.582	0.633
Italy	1.761	1.259	0.786	0.661	0.709	0.677
Korea	1.762	1.617	0.818	0.812	0.840	0.784
Latvia	1.766	1.495	0.797	0.560	0.526	0.629
Lithuania	1.766	1.495	0.797	0.627	0.580	0.642
Mexico	1.768	1.629	0.810	0.741	0.741	0.819
Netherlands	1.761	1.259	0.786	0.553	0.626	0.609
New Zealand	1.770	1.629	0.827	0.617	0.644	0.668
Norway	1.761	1.620	0.821	0.751	0.646	0.733
Portugal	1.766	1.495	0.797	0.648	0.706	0.671
Slovak Rep	1.757	1.125	0.770	0.622	0.626	0.677
Slovenia	1.766	1.495	0.797	0.574	0.762	0.625
South Africa	1.762	1.618	0.818	0.745	0.699	0.709
Spain	1.766	1.495	0.797	0.590	0.637	0.609
Sweden	1.768	1.629	0.810	0.651	0.779	0.631
Switzerland	1.762	1.617	0.820	0.670	0.655	0.646
UK	1.766	1.495	0.797	0.580	0.625	0.637
US	1.766	1.495	0.797	0.544	0.566	0.539
Average	**1.764**	**1.480**	**0.801**	**0.653**	**0.681**	**0.679**

Empirical Results

Table 4: Net AR(1) Coefficients and Standard Errors

Country Group→	High Income			Upper Middle Income			Lower Middle Income			Lower Income		
Series[1]	*Y*	*C*	*I*	*Y*	*C*	*I*	*Y*	*C*	*I*	*Y*	*C*	*I*
Annual Data												
Number of Countries	30			16			19			5		
Average of $(\beta^f - \beta^h)$[2]	0.23	0.20	0.24	0.22	0.27	0.24	0.23	0.26	0.22	0.27	0.28	0.30
Average of $(\sigma^f - \sigma^h)$[3]	0.02	0.02	0.05	0.03	0.03	0.07	0.02	0.03	0.06	0.02	0.03	0.08
Countries *not* passing *Z*-test at 10% for H_0 : $\beta^f - \beta^h = 0$	11	11	9	7	9	6	8	10	6	3	3	2
Quarterly Data[4]												
Number of Countries	28			4			1			0		
Average of $(\beta^f - \beta^h)$	-0.02	-0.02	-0.03	0.00	0.00	0.02	-0.01	0.07	-0.00	-	-	-
Average of $(\sigma^f - \sigma^h)$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00			
Countries *not* passing *Z*-test at 10% for H_0: $\beta^f - \beta^h = 0$[4]	0	0	0	0	0	0	0	0	0	-	-	-

Notes: 1. Y, C and I denote detrended income, consumption and investment series. 2. The average of the net difference in the AR(1) coefficients where superscript f and h denote fully modified HP filter and HP filter respectively. 3. The average of the net difference of the standard deviation of detrended series, where σ^f and σ^h are standard deviation of cyclical component estimated by fully modified HP filter and HP filter respectively.4. AR(1) coefficient equality tests.

Empirical Results

Table 5: Net Unconditional Correlations

Country Group→	High Income		Upper Middle Income		Lower Middle Income		Lower Income	
Pairs[1]	Y-C	Y-I	Y-C	Y-I	Y-C	Y-I	Y-C	Y-I
Annual Data								
Number of Countries	30		16		19		5	
Average of $(\rho_i^f - \rho^h)^2$	0.04	0.03	0.02	0.02	0.10	0.06	-0.02	0.00
Countries *not* passing *Z*-test at 10% for H^0: $\rho_i^f - \rho^h = 0$	21	12	9	8	11	9	3	2
Quarterly Data								
Number of Countries	28		4		1		0	
Average of $(\rho_i^f - \rho^h)$	-0.04	-0.03	0.01	0.00	0.04	-0.02	-	-
Countries *not* passing *Z*-test at 10% for H^0 $\rho_i^f - \rho^h = 0$[3]	18	16	2	3	0	0	-	-

Notes: 1. Y-C and Y-I denote unconditional correlations of individually detrended income-consumption and income-investment pairs. 2. The average of net of the correlation coefficients $(\rho_i^f - \rho^h)$ where the correlation coefficient: ρ^f and ρ^h are obtained from wavelet and modified HP filter separately. 3. Correlation equality tests.

Findings

As we know when lambda is exogenously fixed, estimated loss is function of T (and fixed) lambda, the estimated loss for HP filter is independent of actual series and its underlying dynamics. The slight difference in estimated loss for different countries (as reported in column b) is only because of difference in number of observations for each of the country in the Tables. However, when we apply our weighting scheme with fixed lambda (like in HP filter) we see the estimated loss reduces to 0.599 (0.801) compared to 1.734 (1.764) found while we use the weighting scheme of Hodrick and Prescott (1997) in annual (quarterly) real income series.

Thus FMHP of this study is best approach to minimize the EBP in HP type filtering.

Findings

While comparing the individual detrended series analytics we observe that

a) ‘on average’ the difference in AR(1) coefficients of detrended series using two methods (MFHP filter minus the HP filter) is positive across countries, series for annual data while for quarterly data this difference is almost zero,

b) on average difference in the SEs of detrended series obtained by these filters (MFHP minus HP filter) is also positive across series and countries and frequency (especially for annual data) indicating less of cyclical component is left in trend when we extract cycle using FMHP filter, and

c) the AR(1) coefficients of a cyclical part of a time series obtained from two approaches are statistically significantly different from each other across the countries and series for annual data.

Findings

While comparing the unconditional correlation coefficients we observe two important things. First, for annual data set, on the average the point estimates of cross correlation coefficients between the cyclical components extracted by FMHP filter of the income-consumption and income-investment pairs are higher than those between the cyclical components extracted using the HP filter. However, the opposite is true for quarterly data correlations. Second, although the point estimate difference between pair wise correlation coefficients are small for both annual and quarterly data set, most of these differences are statistically significant. For both annual and quarterly data, there are about 60 percent courtiers having statistically significant pair wise correlation difference. This shows that the choice of λ and weighting scheme are also relevant for second order moments of annual series.

Conclusion

Despite its extensive use to extract cyclical component from a macroeconomic time series, end point bias issue of HP filter is well documented in the relevant literature.

We furthered McDermott (1997) Modified HP filter of endogenous smoothing parameter by combing it with an intuitive weighting scheme compared to Bloechl (2014) to solve EPB in HP filtering. We propose to use linear or non linear increase of penalization (whichever minimizes the cumulative loss) to the terminal points while fixing the end point observations to penalize and endogenous end point observations' weights.

Our FMHP outperforms the conventional filters (like HP, BK and CF filter) in a power comparison study. End point performance of our FMHP is specifically evaluated and found best. When we put FMHP filtering to real life test based detrending (of real income, consumption and investment time series of a large number of countries) we find that our FMHP filter significantly lowers the EPB compared to Bloechl (2014) and that FMHP performs better in moments' analytics compared to HP filter.

Policy Implication

EPB contaminates the estimated trend with the cyclical component and thus underestimate the cyclical component during both the recovery as well as recession. It also results in downward biased standard error of the estimated cyclical component. A downward biased standard error of the estimated cyclical component may give impression of a stable economy, and the underestimated cyclical component during booming/receding economy may delay the necessary stabilization measures by economic managers.

With the use of better estimates of the cyclical behavior (with FMHP filtering) of their economies, economic managers will have better knowledge of the state of their economic dynamics and thus will be able to take necessary stabilization measures at right time.

Thank you.