

Monetary Paradoxes of Baby-Sitting Cooperatives

ASAD ZAMAN

Many authors have described and modelled Keynesian effects in a Baby-sitting Cooperative (BSC), which has the underlying structure of a single good barter economy. We construct a simple model of the BSC economy to explore this issue, and find very surprising results. Outcomes depend on agents beliefs about the decision making process of others, as in the Keynesian beauty contest. For some structures of beliefs, money is neutral, while for others, money can have short and long run effects. The value of money can be high, low, or zero, depending purely upon expectational effects. Also, despite the fact that this is a single good economy, partial equilibrium supply and demand analysis do not work as expected. Some equilibria have excess supply, others have excess demand, and none have a match between supply and demand. Furthermore, flexible prices cannot fix this problem. An additional paradoxical property is that excessive trading can take place. Even though all trades are done with mutual consent, some of them decrease welfare, and banning certain types of trade can lead to Pareto improvements. Thus the superficially simple single good barter economy of BSC displays some subtle, complex and counter-intuitive properties.

JEL Classifications: D71, E52

Keywords: Monetary Policy, Keynesian Economics, Sunspot Equilibria, Neutrality of Money

1. INTRODUCTION

Classical economic theory of the time could not explain the Great Depression, nor the prolonged high unemployment which followed. Keynes argued that this was the result of insufficient aggregate demand, which could be fixed by expansionary monetary policy. These ideas became widely accepted, and constituted the basis for monetary policy until the 1970's. Keynesian theories conflict with the neutrality of money, and suggest that expansion of money leads to inflation only under full employment. Keynesian theories gradually fell out of favour following stagflation resulting from oil price shock in the 70's. This led to emergence of alternative macroeconomic theories, as well as a search for micro-foundations for Keynesian economics. This paper provides some micro-foundations for certain Keynesian phenomena in a specialised economy. The model provides surprising insights into the nature of these phenomena.

The Global Financial Crisis of 2007 has led to a renewed interest in Keynesian theories. In particular, Krugman in “The Return of Depression Economics” argued that

Asad Zaman <asad.zaman@alumni.stanford.edu> is formerly Vice Chancellor, Pakistan Institute of Development Economics, Islamabad.

Keynesian ideas remain relevant to understanding contemporary recessions. To motivate this, he has used a real world example of the Capitol Hill Baby-Sitting Cooperative (BSC). According to an analysis by economists who were members of the Cooperative, the BSC suffered from a recession due to a shortage of “scrip”, the currency used to exchange baby-sitting services.

Krugman’s analysis is based on an intuitive and heuristic analysis by Sweeney and Sweeney (1977). However, the BSC is a very simple single good economy, where the sole function of money is to allow for inter-temporal trade. This simplicity allows for a rigorous analytic treatment. Our research was motivated by the idea of analytically validating the intuitive insights of the Sweeneys and Krugman. Is the BSC a Keynesian economy? Can a shortfall of money create a recession in this economy? A simple model which displays Keynesian effects should be useful in building understanding of these phenomenon in more complex situations. Previous analyses of the BSC have come to the following conclusions.

1. In a purely heuristic analysis, Sweeney and Sweeney (1971) argued that the BSC is a Keynesian economy—insufficient money leads to recession, while excess leads to inflation. Krugman uses the Sweeney arguments without further analysis.
2. With motivation similar to ours, Hens, et al. create a mathematical model for the BSC economy. They show that the BSC economy displays Keynesian properties. There is an optimum quantity of money, and too little money leads to recession. They also conduct an experiment which validates their theory, in that the experimental results conform to the theoretical predictions. They suggest that the existence of the optimum quantity of money is due to fixed prices – one unit of scrip can be exchanged only for one unit of baby-sitting.
3. With a somewhat different model and motives, the analytical analysis of Kash, et al. reaches similar results for the BSC economy. They find that increased money supply leads to increased exchange up to a critical limit which is the optimal quantity of money. They find a new phenomenon of a “crash”. Increasing money supply beyond the optimal quantity leads to zero trade as the value of money collapses to zero. Kash, et al. also suggest that the optimal quantity is due to fixed price of scrip.

All authors mention as significant the “fixed price” feature of the BSC economy. But in presence of fixed prices, the existence of an optimal quantity of money, and recession for low money is a triviality. The Keynesian rejection of neutrality of money is not based solely on sticky prices. In this paper, we create a simple model of the BSC economy to investigate the presence of Keynesian phenomena. The model leads to strange and paradoxical results, not available in earlier analyses. We list these results below.

1. The BSC Economy has the Keynesian *Beauty Contest* property. That is, equilibria depend heavily on the beliefs of agents about how other agents will behave. For the sake of definiteness, let us call this “second order expectation”: agents’ expectations about the decision making procedures being followed by other agents. Different types of 2nd order expectations are possible, and lead to

different types of phenomenon. All of the expectation mechanisms explored are “rational” in the sense of being self-fulfilling, creating a justification for believing in their validity.

2. The central question being investigated is: is money neutral in the BSC economy? This has a subtle, complex and perhaps paradoxical answer. In all models, money is “technically” neutral—that is, all levels of money are compatible with the same sets of equilibria. This is true *even though prices are fixed*. At the same time, money is not “expectationally” neutral. At any given level of money stock, coherent expectations about the value of money will be self-fulfilling. In our model, there are three possible coherent and self-fulfilling expectations about money: money is of high value, low value, or zero value. The quantity of money will fail to be neutral if changes in the stock of money affect the expectations about the value of money. If expectations are not affected, then changes in quantity of money have no effect on the equilibria and money is neutral in short and long run.
3. A paradoxical violation of Say’s Law: The BSC economy display several other phenomena which run counter to standard intuitions. Supply creates its own demand in two strong senses. In a single period, supply of baby-sitting is jointly produced with demand, and hence supply creates demand. In addition, all agents balance budgets across time, so that for any single agent, an act of supply is exactly matched by a demand for baby-sitting services at some other point of time. Despite this dual guaranteed match between supply and demand, some equilibria have excess supply, others have excess demand, and none have a match between supply and demand.
4. Breakdown of Partial Equilibrium Supply and Demand Analysis. This phenomenon of mismatch between supply and demand has been noted by many authors, and attributed to the fixity of price—one scrip is worth one half-hour of babysitting. We show that flexible prices cannot resolve this problem. One might expect that the partial equilibrium (PE) Marshallian theory would work in a single good economy. However, we will see that supply and demand cannot be separated as required by PE analysis, and thus the intuitions generated by supply and demand analysis do not hold up.
5. Excessive Trading is Possible. Intuition suggests that trade by mutual consent is always welfare improving, since both parties agree to the trade only under this circumstance. Thus, Kash, et al. argue that the volume of trade is a good indicator of welfare in the BSC economy. In our model, despite mutual consent, trades can be welfare decreasing, and banning certain trades can lead to welfare improvements. Equilibria with lower total trading volume can be superior to situations with higher trading volumes.

All of these paradoxical properties suggest that the surface simplicity of the BSC Economy is deceptive, and hides deep and murky complexities. Although it would be premature to jump to policy implications on the basis of such a simple model, these implications are valid for the BSC economy itself, and are radically different from those suggested by standard economic intuitions. Two of these implications are highlighted below:

1. The value of money can change from high to low and to zero depending purely upon expectations in the BSC economy. It seems likely that this phenomenon will generalise far beyond the simple BSC economy, since multiple equilibria driven by expectations are ubiquitous in monetary models; see for example Evans and McGough (2005). Central Bank responses to speculative attacks on currencies are guided by the intuition that the value of currencies are determined by fundamentals. Thus, speculators cannot win if the fundamentals are sound. Many Central Banks have bet heavily and lost heavily against speculators on the basis of these intuitions. In the BSC economy, the value of money does not depend upon fundamentals, but purely on expectations about the value of money. Thus a speculative attack can succeed just by changing expectations, without any change in the fundamentals. A subtle and complex interaction between fundamentals and the value of money occurs because of the nature of expectations. If everyone believes that fundamentals are relevant to the value of money, then this becomes a self-fulfilling prophecy. Speculative attacks will then take the shape of news about change in fundamentals, regardless of whether or not such change has occurred or whether the changes being described in the news actually matter in the determination of the value of the currency. As long as the news convinces the public, and changes their expectations about the value, the attack will succeed, regardless of Central Bank interventions. This matches empirically the way speculative attacks are conducted and has radical implications for policy in face of such attacks.
2. The Sweeneys and Krugman suggests that low money supply leads to a recession in the BSC economy. In our model, this can happen but has radically different implications from the ones drawn by these authors. First, the expansion of money works through the expectations effect, and so monetary problems are not purely technical. They have a social dimension and work through consensus about the value of money. Second, even though increased money supply may increase the volume of trade, this may actually decrease social welfare. Thus, the so-called recession state, with low volume of trade and high value of money, may actually be superior in terms of welfare to a high volume of trade with low value of money. Again this is in strong conflict with standard economic intuitions.

An important lesson from our model is that choice of a particular equilibrium among a multiplicity of Nash Equilibria requires agents to coordinate plans. The central message of Bicchieri (1997) is that we must go beyond individual rationality, and study how agents actually learn to resolve the coordination problem. Behavioural economics provides us with the possibility of studying such problems, involving how a particular Nash equilibrium is chosen. Duffy (2008) has provided an extensive survey of this literature. Our research suggests that we need to move beyond individual decision making to study collective decision making in problems with multiple Nash equilibria.

2. A MODEL OF A BSC ECONOMY

The BSC is a single good economy. In each period, agents (families) can either produce the good (offer baby-sitting services) or consume it (receive baby-sitting

services). Unlike Hens, et al. (2007) and Kash, et al. (2007), we consider a finite horizon economy which terminates at a fixed and known endpoint in time. We also assume zero discount rate. These simplifying assumptions bring out clearly a core feature of the BSC economy which is obscured in the infinite horizon treatments. If prices are fixed in monetary terms, then the budget constraint means that the number of goods produced must exactly equal the number of goods consumed. This is valid both cross-sectionally and across time. That is, at every point in time, agents who produces services must exactly match in number the agents who consume. Also, every family must balance its lifetime budget by consuming services exactly equal to its production.

Assume that all families start out with an initial allocation of M_0 units of money. Money—also called scrip in this context—must be used to buy services, and is earned by offering services. However, on the terminal date T , every family must return the full amount of the initial allocation. This means that families must maintain a balanced budget over the lifetime of the BSC economy. The initial allocation is only a device to allow them to borrow from future earnings. The initial money stock M_0 can be made arbitrarily large, to prevent any artificial upper limit to this borrowing. Borrowing from future is automatically constrained by the maximum lifetime earnings remaining to terminal date, which declines to zero as we approach terminal date.

The basic problem is similar to that of inter-temporal consumption smoothing. The utility from consuming and producing babysitting services vary at random with time. Families would like to consume when the need is High, and produce when the cost is Low. For simplicity, we consider three states of need for babysitting service: Z(ero), L(ow), and H(igh). Families have three actions available: {P(roduce), C(onsume), I(dle)}. As a benchmark, we suppose that $U(P|Z)=U(C|Z)=U(I|Z)=0$; the utilities from idling, producing or consuming in state Z are all equal to 0. Also, $U(C|H) > U(C|L) > 0=U(C|Z)$ while $U(P|H) < U(P|L) < 0=U(P|Z)$; utilities of consuming baby-sitting services are ranked High, Low and 0 in states H, L and Z, while costs of providing baby-sitting services are in reverse order in these three states. Key assumptions regarding intertemporal swaps for any single agent are as follows:

- $U(C|H)+U(P|H) < U(I|H)+U(I|H) \ll 0$. Agents do not want to buy services in state H if they have to pay for them in state H. They would rather stay idle in both states. However, idling in state H, or not having baby-sitting services in a situation of high need, is a very poor outcome.
- $U(C|L)+U(P|L) < U(I|L)+U(I|L) < 0$. The same is true for state L. Idling in both periods is preferable to producing and consuming.
- $U(C|H)+U(P|L) > 0$. Consuming services in state H and paying for them in L is preferable to staying idle in both states.
- $U(C|H) > U(C|L) > 0$ guarantees that agents are happy to buy services in states H and L if they can pay for them by selling in state Z.

To understand the results to follow, it is useful to bring out certain implications of our assumptions about the utilities. We will assume that individuals maximise the sum of undiscounted expected utilities, to preserve the underlying one-for-one barter structure of the trading. Note that expected utilities are cardinal; the actual numbers matter. Nonetheless, our results will be valid for a large range of numbers satisfying the

following qualitative considerations. First, it is very important to be able to get baby-sitting in the high need state: $U(C|H) \gg 0$. We will refer to failure to get service in state H as a “crisis”: $U(I|H) \ll 0$. Having to provide services in state H is even worse: $U(P|H) < U(I|H)$. What happens in state L is of lower order of magnitude in comparison. That is, if the utility numbers are in the thousands for state H, they are in tens for state L. It is pleasant to get services in state L and an annoyance to provide services in state L. Not getting service in state L is a minor nuisance. To a first approximation, the efficiency of any system for the baby sitting economy can be gauged in terms of its ability to prevent crises. If all high need demands for baby-sitting are fulfilled, then the system is functioning efficiently. All agents like to consume in state H, and to produce in state Z to pay for this consumption. Dynamics are driven by what happens in state L. If agents do not get enough income from sales in state Z to satisfy their H(igh) need demands, then they will produce in state L to finance consumption in state H. However, if they have sufficient income to cover their H demands, they will use the excess income to purchase baby-sitting services in state L. Thus in state L, agents can be either producers or consumers depending on market conditions.

Demand and supply depend on randomly generated states of need (H,L,Z) and hence need not be equal in any period of time. We assume that market will be cleared via randomised rationing, as in Hens, et al. (2007). That is, if there is excess supply, the demand will be distributed randomly over the suppliers; some will be chosen to supply, while some will remain idle, with identical probabilities for all agents. Excess demand is also handled similarly by randomised rationing of available supply. Crucial parameters are the probabilities $p(B)$ and $p(S)$ of being able to Buy or Sell baby-sitting services. If there is excess supply, then $p(B)=1$ while $p(S)$ is the ratio of the number of agents who are demanding baby-sitting services to the number of agents who are offering baby-sitting services. In cases of excess demand, this is reversed: $p(S)=1$ while $p(B)$ is the reverse ratio. Hens, et al. (2007) and Kash, et al. (2007) both assume that the agents know these probabilities. However, neither discusses how the agents might learn these probabilities.

How agents calculate these probabilities ($p(B)$ and $p(S)$), which are required for rational decision making, is central to the operations of the BSC economy. Future probabilities cannot be calculated solely on the basis of observable variables of aggregate demand and supply. Every agent must know how other agents are making these calculations in order to arrive at an accurate estimate. This is why second order expectations are crucial to rational decision making and equilibria in the BSC economy. To demonstrate this, we work with two different models for second order expectations. One is an oracular model, while the other is a threshold model. We first describe the Oracular expectations model, which is simpler of the two.

3. ORACULAR EXPECTATIONS

To demonstrate the existence of sunspot equilibria, we assume the existence of a Fama-Oracle which forecasts the probabilities $p(S)$ and $p(B)$. As long as all agents believe in the Fama-Oracle, these forecasts always turn out to be accurate and therefore create rational expectations.

We assume that the Fama-oracle announces the probabilities $p(B)$ and $p(S)$, and that it is common knowledge that everyone believes in the Fama-oracle. Note that this is

an assumption about a particular structure of second order expectations. In addition, decisions require knowledge of the probabilities $p(Z)$, $p(L)$ and $p(H)$ of the states Z, L and H. We assume that these three probabilities are known to all and the same for all agents, as well as across time. We also assume that nature generates these states in such a way that the actual proportion of the states Z,L,H across agents is exactly equal to the these three probabilities. Furthermore, over the lifetime of an agent, the actual frequency of occurrence of the three states is also matched to the theoretical probabilities of each of the three states Z, L and H. This assumption simplifies calculations and avoids peripheral complications, without affecting the central results.

Knowledge of these five probabilities ($p(B), p(S), p(H), p(L), p(Z)$) allows us to compute the agents' maximising strategies. We assume that $p(H) < \frac{1}{2}$ and also $p(Z) < \frac{1}{2}$. These inequalities imply that $p(H) < p(Z) + p(L)$ and also $p(H) + p(L) > p(Z)$.

3.1. Three Rational Expectations Equilibria

We will now show that the Fama-oracle can create three rational expectations equilibria.

Excess Supply Equilibrium (High Value of Money): Assume the oracle announces that for the foreseeable future there will be excess supply so that probability of being able to buy baby-sitting is unity: $p(B)=1$. Sellers will be rationed, with $p(S) = p(H) / [p(L)+p(Z)]$. This will be a self-fulfilling prophecy. Scrip has high value in this equilibrium in the sense that owners of scrip are guaranteed to be able to buy baby-sitting services

Excess Demand Equilibrium (Low Value of Money): Assume the oracle announces that for the foreseeable future there will be excess demand, so that $p(S)=1$. Demanders will be rationed, and will succeed in buying with probability $p(B)=p(Z)/[p(H)+p(L)]$. This will be a self-fulfilling prophecy. Scrip has lower value in this scenario since owners of scrip can buy baby-sitting services only with probability $p(B)<1$.

Zero-Trade Equilibrium (Zero Value of Money): If the Fama-Oracle announces that the Gods are angry, and the value of money will be zero from now on, this too will be a self-fulfilling prophecy. In this case $p(B)=p(S)=0$, and no one can buy or sell baby-sitting services.

Proofs: The Zero-Trade Equilibrium is trivial. The other two cases can be proven as follows:

Excess Supply Equilibrium. Suppose the oracle announces that $p(B)=1$, and $p(S)=p(H)/[p(L)+p(Z)] < 1$. Then all agents maximise lifetime utility by always buying services in state H, and by always offering to sell in states L and Z.

Proof: To a first order approximation, agents maximise lifetime utility by avoiding crises. So we start by assuming that all agents always demand baby-sitting services in state H. To finance these purchases requires an income of $p(H) \times T$; the proportion of time agents are in state H, times the total time horizon T. Agents maximise lifetime income by always offering to sell in states L and Z. This generates income equal to $p(S) \times [p(L) + p(Z)] \times T$ because offers to sell are completed with probability $p(S)$. This is exactly equal to $p(H) \times T$, the income needed to purchase services in state H. Agents generate maximum possible income, which is exactly enough to meet their high priority demands for baby sitting—no crises. Any change in any decision (offers to buy or sell) will result

in crises, either because of failure to request services in state H, or insufficient budget to buy services.

Excess Demand Equilibrium: Suppose $p(S)=1$, and $p(B)=p(Z)/[p(H)+p(L)] < 1$. Then all agents maximise lifetime utility by always offering services in state Z, and always requesting services in states H and L.

Proof: Agents always succeed in selling in state Z, so their lifetime income is $p(Z)$. Income needed to purchase their lifetime demand of services in states L and H is $p(B) \times [p(H)+p(L)] = p(Z)$. Offering services in state Z is always optimal. Since this creates exactly enough income to allow them to request services in all states L and H, this is necessarily an optimal sequence of decisions. Failure to request services will result in surplus, unutilised income.

Technical Note: We take a large finite T, and ignore complications that would arise near the terminal date. Our treatment can be made rigorous by using a limit process as both the number of agents N, and the time period T approaches infinity. This would be a formalisation of the Ramsey-Weizsacker overtaking criterion. There are many other ways to resolve the problem, but our main results are robust to minor changes in how we handle the complications near the terminal date. Our treatment provides conceptual clarity with a minimum of mathematics.

Discussion: Note that the quantity of money is irrelevant in these equilibria, as long as M_0 is sufficiently large to prevent constraints on borrowing from the future. Another option is to allow agents to borrow from each other, or from the central authority. In either case, the quantity of money will be irrelevant to short run and long run equilibrium. Thus, contrary to the analyses of earlier authors, there is no optimum quantity of money in the BSC model under Oracular Expectations. However, we can create monetary effects if we link oracular forecasts to the money supply. First, we give an example to illustrate how rational expectations can create an illusion of causality between money and economic outcomes.

Suppose that the Fama oracle announces that there will be excess supply on days when the Air Quality Index (AQI) is above 50, and excess demand when AQI is less than 50—it appears perfectly plausible that people would want to go out when the pollution index is low, and to stay home otherwise. Under our model assumptions, this would also create a self-fulfilling prophecy. Thus, people could come to believe, based on solid empirical evidence, that the AQI has a causal effect on demand and supply of baby-sitting when in fact it has zero effect. To be more precise, the causal effect is created by the belief in the existence of the effect (via the Fama-Oracle intervention).

Similarly, if the Fama-Oracle announces that there will be excess supply if aggregate money stock (known to all) is greater than some threshold M^* , and excess demand when it is less, this too will be a self-fulfilling prophecy. Thus monetary effects can be created if the Fama-Oracle chooses to create them.

4. THRESHOLD EXPECTATIONS

Both Hens, et al. (2007) and Kash, et al. (2007) study equilibria in threshold strategies, which is a different assumption about how agents behave in response to changes in money stock. To show that the BSC economy has the Beauty Contest property, we now study this alternative assumption about second order expectations. To

get the desired equilibria, it must be common knowledge that all agents behave using the strategies described below.

4.1. Threshold Strategies

Kash, et al. (2007) and Hens, et al. (2007) find Nash equilibria such that agents play threshold strategies. In our model, we can characterise such strategies as follows. Agents in state H always offer to buy, while agents in state Z always offer to sell. Suppose an agent holds a stock of money M . In state L, there exists threshold values M^{**} and M^* such that agents buy if $M > M^{**}$ and sell if $M < M^*$. Between the two values, agents stay idle. If it is common knowledge that all agents play the same threshold strategy, then the agents can co-ordinate their beliefs about what will happen.

Let $m(a,t)$ be the money endowment of agent a at time t . In going from time period t to $t+1$, the sellers' stocks will increase by 1, buyers' stocks will decrease by 1, and those who stay out of the market will remain at the same level. These transition probabilities create a Markov chain which has a limiting stationary distribution, exactly as demonstrated by both Hens, et al. (2007) and Kash, et al. (2007). Thus, for sufficiently large values of t , there exist probabilities $p^*(j)$ for $j=0,1,2,\dots$ such that agents have money stock $m(a,t)=j$ with probability $p^*(j)$. These lead to three stable probabilities $p(BS)$, $p(BB)$ and $p(BD)$ of budget surplus ($m(a,t) > M^{**}$), budget balance ($M^{**} \geq m(a,t) \geq M^*$), and budget deficit ($m(a,t) < M^*$):

$$p(BS) = \sum_{j > M^{**}}^{\infty} p^*(j), p(BB) = \sum_{M^{**} \geq j \geq M^*}^{\infty} p^*(j), p(BD) = \sum_{j < M^*}^{\infty} p^*(j),$$

Once the Markov chain reaches stationarity, the proportion of buyers is $p(H)+p(BS)$ $p(L)$, while the proportion of sellers is $p(Z) + p(BD)$ $p(L)$. We ignore the initial period required for the Markov chain to reach stationarity, and calculate equilibria under the assumption of stationarity—this corresponds to the analysis of Hens and Kash, et al. who assume that the game starts in an equilibrium position. There is no harm in this assumption, since we are just illustrating some phenomena which would occur near the middle of the game. We will now show that the Nash equilibria of the threshold economy are the same as those of the oracular economy.

Proposition 1: In equilibrium, all agents in state L must play the same strategy – either buy or sell. Both of these possibilities form Nash equilibria, which are the same as the excess supply and excess demand equilibria of the Oracular economy.

Proof: First consider a case where every agent can be in Surplus or in Deficit with positive probability: $p(BS) > 0$ and $p(BD) > 0$. With these values for M^* and M^{**} , all agents will find themselves on both sides of the market in state L during their lifetime. This is because the Markov chain is irreducible, and all states can be reached from all other states. Thus any agent can gain by adjusting the thresholds M^* and M^{**} in such a way that one of these two probabilities is reduced to zero. This will eliminate intertemporal swaps of buying and selling in state L, leading to improved welfare. This means that values of M^* and M^{**} which lead to $p(BS) > 0$ and $p(BD) > 0$ cannot represent an equilibrium.

Next suppose the thresholds M^* and M^{**} are such that probability of budget surplus or balance are positive ($p(BS) > 0$ and $p(BB) > 0$) but there is no probability of a deficit: $p(BD) = 0$. If a single agent shifts thresholds to make $p(BB)$ smaller and $p(BS)$

larger, she will benefit from this adjustment. The actions of a single agent do not affect the aggregate probabilities $p(B)$ and $P(S)$ of purchase and sale, so she will be able to buy additional services in state L, improving her payoff. If all agents make these adjustments, the stable point of this adjustment process will be $p(BS)=1$ and $p(BB)=p(BD)=0$, which is the excess supply equilibrium. A similar arguments shows that $p(BS)=p(BB)=0$ while $p(BD)=1$ also leads to equilibrium.

4.2. Interpretation of Rational Expectations Equilibria

Many authors interpret threshold strategies as follows. Agents seek to have a minimal level of reserves, to provide them with a cushion against a sequence of unanticipated high priority needs. This interpretation is also supported by the actual experiences of the baby sitting cooperative as well as experimental evidence provided by Hens et. al. However, this interpretation is not fully satisfactory. This is because the economy as a whole has a fixed amount of money, and savings of one is dissavings of another. So as a group, members of the BSC cooperative cannot achieve the goal of higher savings. Also, all members play balanced budget strategies. So their money holdings form a random walk centred on initial holdings—savings cannot increase systematically. The goal of increasing reserves is an illusion, both individually and collectively. How can we expect high levels of rationality and maximisation from our agents, if they fail to realise something as simple as this?

The sunspot interpretations provide an explanation. If one agent is a skeptic, but thinks that others will believe the oracle, then it is still optimal for her to follow the Nash strategy. As discussed, the equilibrium has properties of the “beauty-contest”. Even if all agents are skeptics, but consider that other agents will compute strategies under the assumption that all others are believers, the same Nash equilibrium will result. In exactly the same way, agents can co-ordinate on threshold strategies without believing that these are good strategies, if they think that everyone else will be reasoning in this way. Hens, et al. (2007) provide experimental evidence to suggest that subject do in fact follow threshold strategies, which would partly explain the experiences of the original baby-sitting cooperative.

5. LESSONS FROM THE BSC ECONOMY

Despite the surface simplicity of a single good, one-for-one barter economy, analysis of the BSC economy leads to deep, subtle, and counter-intuitive results. We summarise these results, and discuss their implications.

5.1. The Neutrality of Money

What comes out very clearly from the analysis of the BSC economy is that money is at least partly a “social construct”. It derives value from our mutual agreement about its value. Thinking about the Fama-oracle as a mechanism to arrive at consensus, money have can have high, low or zero value according to our mutual agreement on one of these values. At the same time, money is not purely a social construct. The underlying structures of supply and demand determine the value of money in the two non-trivial equilibria. Thus the value of money emerges by the interaction of social norms with the economic environment. This seems to be well understood by central bankers and

treasuries who use a combination of confidence building measures such as transparency together with technical measures such as setting interest rates and open market operations to control the value of money.

Is the BSC a Keynesian economy in the sense that money is not neutral? Our analysis shows clearly that the answer is yes and no. If the agents coordinate second order expectations on the assumption that money will have high value, then the excess supply equilibrium will result. This will not change regardless of how much money there is in the system, as long as borrowing is allowed. Thus money will be neutral in short and long run. However, if agents believe that increases in money will be interpreted by other agents to imply a decrease in the value of money, then money will fail to be neutral. An increase in money supply will lower the value of money, but only because everyone believes it will do so.

5.2. Supply, Demand, and Flexible Prices

A key lesson from all previous analyses of the BSC economy is that there exists an optimal quantity of money in the BSC economy. As we have seen, this result is tied to unstated implicit assumptions about second order expectations of the agents. Under certain types of second order expectations, money is neutral, while under others it is not. In fact, failure of neutrality is surprising in standard models which are homogenous in money and prices. If a certain set of prices and money stocks (p^* , m^*) leads to efficient outcomes, then $(\lambda p^*$, λm^*) will also lead to the same efficient outcomes, for any positive scale factor λ . As long as prices are flexible, there can be no optimum quantity of money. This is why Sweeney & Sweeney (1977) and other authors have argued that it is fixed prices which lead to shortages and rationing. A system of flexible prices would lead to clearing of markets, and to non-existence of an optimal quantity of money.

Unfortunately, in our BSC model, this does not work as expected. For the sake of clarity, consider a specific case where $p(Z)=p(L)=p(H) = 1/3$. All three states are equally likely. Consider the excess demand equilibrium in which all agents in states Z are sellers and all agents in states L and H are buyers. In principle, we should be able to fix the problem by raising the price of baby-sitting to reduce excess demand. Consider therefore doubling the price—the cost of baby-sitting is two units of scrip. Those who are buying must pay two units, while those who are selling will receive two units of scrip. Now note that every agent is on both sides of the market at different points in time. Today as buyers, they have to pay double, but tomorrow as sellers, they will receive double price. So the budget constraint does not change. An agent will pay two units in when he is a successful buyer in states H and L, and receive $2 \times 1/3$ in the state Z. Agents still have balanced budgets and therefore the excess demand will persist at any scrip price.

Our intuition for the idea that price flexibility would resolve mismatch of supply and demand is generated by the Marshallian partial equilibrium analysis. One would expect that the partial equilibrium analysis would work in a single good economy. However, the analysis fails because supply and demand are entangled. Since agents are both suppliers and demanders, a price rise affects both sides simultaneously. Lower demand due to increased prices is exactly offset by the rise in demand due to higher income.

Instead of nominal prices in scrip, we could consider changing the real price of baby-sitting. Now those who buy one unit of baby-sitting services must pay for it by offering two units. Now the agents will plan to buy in state H and sell in states L and Z. If there is no rationing, then these plans would go through. That is, agents can finance the purchases in 1/3 cases of H, by paying in 2/3 cases of L and Z. However, there is now excess supply and the probability of sale will be only $\frac{1}{2}$, so agents will actually foresee a budget deficit. This is a general feature of the BSC economy. The technology is such that in any period of time, sales and purchases occur in pairs, one for one, and must match exactly. Also, across time, every agent must match every sale with a purchase on a 1:1 basis. Thus real prices are technologically fixed and cannot be varied.

5.3. A Paradoxical Violation of Say's Law

Krugman (2011) argues that the BSC economy demonstrates a violation of Say's Law. We note here how surprising and paradoxical this is. This is because this is one model in which we could attach strong expectations to the validity of Say's Law. The technology of baby-sitting is such that every good produced is automatically matched with a consumer – baby-sitting is a two sided transaction. Supply cannot be produced without demand. Furthermore all agents balance their budgets, paying for every unit consumed by one unit produced. Thus every act of supplying babysitting is exactly offset by a demand for babysitting at some other point in time. The logic of Say's law, that production creates its own demand, holds within every time period, and for every agent across time periods. If there ever was a model in which Say's Law holds, then this would be it. However, as we have seen, one of the oracular equilibria of this economy has excess supply, in violation of Say's Law. The other one has excess demand, and there are no equilibria where the two are balanced.

5.4. Efficiency Considerations

The no-trade equilibrium is, of course, highly inefficient. It can easily be seen that both of the non-trivial sunspot equilibria are also inefficient. Of the two, excess supply is usually better because all high priority demands are fulfilled; there are no crises. The inefficiency arises because the supply is proportionally split between low cost and zero cost producers, L and Z. It would be more efficient to have all production done by zero cost producers, with L producers only producing as much as minimally required to fulfil High Priority demands. Here an intervention which bans L producers from selling services could lead to greater efficiency. This is worth examining in detail because of its paradoxical implication that banning a trade done by mutual consent leads to improved welfare for all participants. We will show below that either of the two equilibria – excess demand or excess supply—may be more efficient than the other.

First, consider a situation where $p(H)= 20\%$ $p(L)= 50\%$ $p(Z)= 30\%$. In the excess supply equilibrium, agents in states L and Z offer babysitting services and their offers are accepted with probability 25 percent. In order to pay for their demands in the H state, agents must offer services in both L and Z states to generate sufficient income. Money has value 1, and all high priority demands are fulfilled, so there are no crises. For simplicity, assume there are 100 traders. Then there are 20 trades of baby sitting services in each period. Next consider the excess demand equilibrium. Agents in state Z offer

services and are guaranteed to find a buyer. Agents in states H and L demand services and find a seller with probability $3/7$. There are 30 trades each period, but less than 50 percent of high priority needs for baby sitting are fulfilled and there are large numbers of crises. Thus, we have a high volume of trade (30 trades instead of 20) but lower efficiency. Total cost free supply of baby sitting is 30, and an ideal solution would involve providing 20 to those in state H, and distributing the 10 remaining at random over the 50 agents in state L. Implementing such a solution via an anonymous market mechanism is impossible when the states are not observable. However, social mechanisms involving self-assessment and honest revelations of needs might work. In this example, excess supply is better than excess demand in terms of welfare. However the opposite may be true for other configurations.

For example, suppose that $p(H)=20\%$, $p(L)=30\%$ and $p(Z)=50\%$. In the excess supply equilibrium, 80 percent of the agents in states L and Z will offer baby sitting. These offers will be accepted with probability 25 percent so that one unit of credit will be earned every four periods. This will be just enough to pay for babysitting needs in the High Demand state which occurs once every five periods. There will be no crises, but all Low priority demands will remain unfulfilled while there will remain many zero cost suppliers who are unable to find buyers. The excess demand equilibrium is much better. In this case, the 50 percent of agents in states H and L will demand services, which will be exactly met by the zero cost supply from agents in state Z. By coincidence, the demand and supply are perfectly matched and there is no excess demand, as there would be if $p(H)+p(L)$ was slightly greater than 50 percent.

6. EXPLAINING THE EXPERIENCES OF THE BABY SITTING COOPERATIVE

Understanding an implicit coordination process requires explicit modelling of the learning process followed by agents to arrive at an equilibrium. We suggest one such model which could explain the experiences of the baby sitting cooperative, as described by Sweeney and Sweeney (1971). Assume the $p(H)=25\%$, $p(L)=50\%$ and $p(Z)=25\%$, for the sake of concreteness. All agents start out with initial scrip endowment of 10 units. They all arbitrarily choose thresholds M^{**} and M^* which lead to certain overall probabilities $p(BS), p(BB)$ and $P(BD)$ for the group as a whole. These probabilities determine the supply and demand of babysitting services at arbitrary initial levels. Suppose that these arbitrary initial choices lead to 20 percent of agents in state L being sellers, 20 percent being buyers and 10 percent remaining idle. Then overall demand is 45 percent which is balanced by overall supply of 45 percent, while 10 percent of the families remain idle. Inefficiency is caused by the fact that agents in state L are both buyers and sellers – all utilities would improve if they would just stay out of the market.

What are the signals that agents receive that they can do better, and cause them to adjust strategies towards equilibrium? The buyers are buying in state L because they perceive themselves as rich relative to an arbitrarily chosen threshold M^{**} – they think they have enough credit to be able to buy services in both states H and L. The sellers have set a high threshold M^* , and perceive themselves as poor: they don't have enough credit to pay for potential crises, and are accumulating money. Attitudes towards risk would have a strong impact on these initial choices of thresholds. Since states Z and H

cancel in terms of monetary change, the buyers will decumulate money once every 5 periods (20 percent probability) and sellers will accumulate money with the same frequency. Assume $M^{**}=M^*$ for simplicity. Then some buyers will transition to poverty and will become sellers. At the same time some sellers will transition to richness will become buyers. This state of affairs could persist for quite some time.

Assume that a psychological shock is generated by a crisis. As the random walk in money holdings takes some proportion of agents close to 0, they realise that they should hold more money in reserve to prevent crises and switch to a higher threshold M^* . These agents will now become sellers in state L. As this process continues, the proportion of sellers will rise and the proportion of buyers will decline. Starting from a balanced supply and demand, it will move to a position of excess supply. To determine suitable threshold levels, an agent could calculate as follows. My high priority needs in the next 100 periods are 25 percent of $100 = 25$. My maximum revenues in the next 100 periods will be $p(S) \times [p(L)+p(Z)] \times 100 = p(S) \times 75$. In initial periods of the economy when demand and supply were balanced, agents were quite safe. When $p(S)=1$, they have earning capacities of 75, and high priority demand for only 25. However, as some agents experience crises and become sellers only in state L, the probability $p(S)$ will start declining, leading to loss of potential income—when $p(S)$ is 50 percent then the maximum potential earnings is reduced to 37.5 from 75. This will again induce agents to increase thresholds to a higher safe level. This adjustment process will terminate when all agents become sellers with 100 percent probability in state L. In this case $p(S)=1/3$ which leads to an exactly balanced budget from 75 offers to sell and 25 offers to buy. Now we have a steady state equilibrium in which the supply is three times that of demand. Economists consider that the economy has fallen into recession. In fact, as we have seen, this is a good equilibrium in that there are no crises. Nearly all high priority needs are being met. A small proportion of agents will run out of money and will experience crises, but this problem can be solved by extending them credit.

What happens when money is injected into this economy? To answer the question, we must posit a model for how agents form expectations about the behaviour of others. If for example agents believe that nothing has changed, then nothing will change, because everyone is at a stable Nash equilibrium, maximising utility subject to a budget constraint. However, change can happen if agents reason as follows. Now I have enough money reserves that I need not fear a crisis. I no longer need to sell services in state L, I can choose to buy instead, raising my welfare. As this type of thinking diffuses through the agents, demand will increase, supply will fall and $p(S)$ will start rising. At some point, it will reach back to $p(S)=1$, when half of the agent in L are rich and the other half are poor. Note that richness and poverty is a state of the mind in this story, not an objective reality. Also note that as $p(S)$ rises, agents can observe and calculate that their maximum potential earnings are increasing and therefore they are becoming richer. This will accelerate the process of change. Somewhere in the middle of the transition from excess supply to excess demand, there will occur a “golden age” where demand and supply are perfectly balanced, and the volume of trade is at a maximum. However the process will inexorably continue on past this point and go on to the excess demand equilibrium. How long the transition takes depends on details about how agents adjust expectation in changing circumstances. Economists will interpret the excess demand as being due to over-supply of money. This story corresponds closely to the one narrated by the Sweeneys of the actual experience of the baby sitting economy.

7. IMPLICIT VERSUS EXPLICIT COORDINATION

Economists who have analysed the recession in the BSC economy have poked fun at lawyers who attempted to motivate families to go out more often. The lawyers were trying to find a social solution to what was apparently a purely technical monetary problem. Our analysis shows that the failure of lawyers was their lack of persuasiveness (Capitol Hill credibility gap?), and not a wrong approach. The standard approach used in theoretical and experimental economics relies on implicit co-ordination. Making iterative adjustments to optimal responses mimics a natural learning process of implicit coordination. In presence of multiple equilibria, the outcome depends on arbitrarily chosen initial values. As in the “continental divide” game (see Camerer, 2011), Section 1.2.2), small changes in initial conditions can lead to large differences in outcomes. However, once the equilibrium is reached, it will be “sticky”—it will require group effort to change it, since no individual can benefit from shifting from the Nash equilibrium strategy.

An explicit coordination process would involve all the members of the group sitting together to achieve consensus on a common desired outcome. When this is possible, it is clearly more rational to make a conscious choice of a particular equilibrium, rather than letting it be determined by some arbitrary initial choices, coupled with default assumptions about decision making strategies of members of the group. In this context, it is important to note that groups can often achieve consensus on outcomes which are not Nash Equilibria—for instance on the strategy of 100 percent cooperation in the prisoners dilemma. The standard analysis of games is based on Savage’s small world assumption, which studies each problem in isolation. In a community where there is substantial interaction outside of any particular game, this assumption does not hold. A reputation for honesty, and for fulfilling commitments is extremely valuable, and lack of it is extremely harmful in many social interactions. Thus human beings routinely fulfil commitments, even at personal cost, contrary to game theoretic assumptions based on studying the game in isolation as the sole venue of interaction. Once these habits of character are acquired, they are adhered to even in situations of interactions with complete strangers, not belonging to the original community. This means that “cheap talk” can achieve cooperative outcomes even when these are not based on individually maximising strategies. A summary of literature on cooperation by Dawes and Thaler (1988) states that “... the analytically uncomfortable (though humanly gratifying) fact remains: from the most primitive to the most advanced societies, a higher degree of cooperation takes place than can be explained ... (by selfishness)”.

8. CONCLUSIONS

We have already discussed how lessons derived from our model of the BSC model conflict with many of those drawn by other authors. Our model sets up the BSC economy as a single good economy which is bartered across time. Money does not play any real role except as an accounting device. As long as agents can borrow against future earnings, the quantity of money and monetary policy are irrelevant. Also, because of the 1:1 nature of the barter, price flexibility makes no difference to the outcomes.

A deeper lesson is the following. In models involving trades over time, it is crucial to model expectations regarding the future. Failure to model expectations means leaving an essential element of the model unspecified. Our BSC model illustrate what Bicchieri (2012) has argued at book length; that rationality assumptions are not enough to achieve co-ordination. Rather, we must consider how agents learn about each other, and how they arrive at a co-ordinated equilibrium. The invisible hand paradigm, taken as a fundamental organising principle, suggests that a suitable system of prices will efficiently de-centralise decisions. Agents need only consider their own separate optimisation problem, and this will produce optimal social results. However, in cases of multiple equilibria, this insight is not valid. Rather, the agents must cooperate and agree upon a solution which is beneficial to all. This can be done implicitly, via the iterative choices of best response Nash strategies. However the resulting equilibria can be highly inefficient, and can vary dramatically depending on arbitrary initial points. It will be better to co-ordinate strategies and beliefs explicitly, after considering the relative efficiency of different choices. Efforts at explicit coordination may involve designing institutions and invoking or creating appropriate social norms, which is a different paradigm from the standard individual utility maximisation in isolation.

REFERENCES

- Ariely, Dan (2008). *Predictably irrational*. New York: Harper Collins.
- Bicchieri, Cristina (1997). *Rationality and coordination*. Cambridge: Cambridge University Press.
- Bicchieri, Cristina (2006). *The grammar of society: The nature and dynamics of social norms*. Cambridge: Cambridge University Press.
- Camerer, Colin (2011). *Behavioural game theory: Experiments in strategic interaction*. Princeton University Press.
- Cass, David & Shell, Karl (1983). Do Sunspots Matter? *Journal of Political Economy*, 91(21), 193–228 .
- Dawes, Robyn M., & Thaler, Richard H. (1988). Anomalies: Cooperation. *The Journal of Economic Perspectives*, 187–197.
- Duffy, John. (2008). Macroeconomics: A survey of laboratory research. In Kagel and A. Roth (eds). *The Handbook of Experimental Economics* Vol. 2 .
- Evans, George W., and McGough, Bruce (2005). Monetary policy, indeterminacy and learning. *Journal of Economic Dynamics and Control*, 29(11), 1809–1840.
- Frey, B., & Oberholzer-Gee, F. (1997). The cost of price incentives: An empirical examination of motivation crowding-out. *American Economic Review*, 87(4), 746–755.
- Hens, T., Schenk-Hoppé, K. R. & Vogt, B. (2007). The great capitol hill baby sitting co-op: Anecdote or evidence for the optimum quantity of money? *Journal of Money, Credit and Banking*, 39(6), 1305–1333
- Kash, Ian A., Friedman, Eric J. & Halpern, Joseph Y. (2007). Optimising scrip systems: efficiency, crashes, hoarders, and altruists. *Proceedings of the 8th ACM conference on Electronic commerce*. San Diego, California, USA: ACM. 305–315.
- Krugman, Paul R. (2000). *The return of depression economics*. WW Norton & Company.
- Krugman, Paul. (2012). *End this depression now!* WW Norton & Company.
- Sweeney, Joan, & Sweeney, Richard James (1977). Monetary theory and the great capitol hill baby sitting co-op crisis: comment. *Journal of Money, Credit and Banking*, 86–89.