Alternative Capital Asset Pricing Models: A Review of Theory and Evidence

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INTRODUCTION

The proposition that a well-regulated stock market renders a crucial package of economic services is now widely accepted in financial economics. The various important functions of stock exchange include provisions for liquidity of capital and continuous market for securities from the point of view of investors. From the point of view of economy in general, a healthy stock market has been considered indispensable for economic growth and is expected to contribute to improvement in productivity. More specifically, the indices of stock market operations such as capitalisation, liquidity, asset pricing and turn over help to access whether the national economy is proceeding on sound lines or not. In addition to free and fair-trading the stock market can assure and retain a healthy market participation of investors besides improving national economy. In addition there are well-documented potential benefits associated with foreign investment in emerging markets [Chaudhri (1991)]. A major factor hindering the foreign investment in these markets is lack of information about characteristics of these markets especially about the price behavior of equity markets of these countries.

An efficient performance of pricing mechanism of stock market is a driving force for channeling saving into profitable investment and hence, facilitate in an optimal allocation of capital. This means that pricing mechanism by ensuring a suitable return on investment will expose viable investment opportunities to the potential investors. Thus in stock market, the pricing function has been considered important and a subject of extensive research. In the literature behavior of stock market has been studied by employing asset pricing models such as capital asset pricing model (CAPM), the conditional CAPM, and the arbitrage pricing theory (APT), Merton (1973) intertemporal CAPM and Breeden (1979) version of consumption based CAPM.

The main objective of this study is the review of the conceptual framework of asset pricing models and discusses their implications for security analysis. The first two parts (a) and (b) in sections one of the study are devoted to the theoretical derivation of equilibrium model, usually referred to as capital asset pricing model (CAPM). This model was developed almost simultaneously by Sharpe (1964), Treynor (1961), while Lintner (1965) and Mossin (1966) and

Black 1972) have extended and clarified it further. The variation through time in expected returns is common in securities and in related in plausible ways to business conditions. Therefore modified version of the asset-pricing model, known as conditional capital asset pricing model (CCAPM) is presented in part (c) of section one. An alternative equilibrium asset-pricing model called the arbitrage asset pricing theory (APT) was developed by Ross (1976). The fundamental principles underlying the arbitrage prong theory are discussed in part (d) of section one. In section two the literature review is given and implications of the evidence are also discussed. The critical analysis of the theoretical empirical model is presented in section three. The last section concludes the study.

1. REVIEW OF THEORETICAL LITERATURE

The capital asset pricing model has a long history of theoretical and empirical investigation. Several authors have contributed to development of a model describing the pricing of capital assets under condition of market equilibrium including Eugene Fama, Michael Jensen, John Lintner, John Long, Robert Merton, Myron Scholes, William Shaepe, Jack Treynor and Fischer Black. For the past three decades mean variance capital asset pricing models of Sharpe-Lintner and Black have served as the corner stone of financial theory. Another important theory is APT, which is based on similar intuition as CAPM but is much more general. The following parts (a), (b), (c) and (d) presents the theoretical review of these two models.

(a) Capital Asset Pricing Model: Sharpe-Lintner Version

The Sharpe-Lintner model is the extension of one period mean-variance portfolio models of Markowitz (1959) and Tobin (1958), which in turn are built on the expected utility model of von Nuemann and Morgenstern (1953). The Markowitz mean variance analysis are concerned with how the consumer-investor should allocate his wealth among the various assets available in the market, given that he is one-period utility maximiser. The Sharpe-Lintner asset-pricing model then uses the characteristics the consumer wealth allocation decision to derive the equilibrium relationship between risk and expected return for assets and portfolios.

In the development of capital asset pricing model simplifying assumption about the real world are used in order to define the relationship between risk and return that determines security prices. These assumptions are, (a) all investors are risk-averse individuals, who maximise the expected utility of their end of period wealth, (b) the investors are price takers and have homogenous expectations about asset returns that have joint normal distribution, (c) there exist a risk-free asset such that investor may borrow or lend unlimited amounts at the risk-free rate, (d) the quantities of asset are fixed, also all assets are marketable and perfectly divisible, (e) asset markets are frictionless and information is costless and simultaneously available to all investors, and (f) there are no market imperfections such as taxes, regulations, or restrictions on other sellings.

The development of the asset pricing model begins with the description of market setting within which equilibrium must be established. It is assumed that all production is organised by firms. At the beginning of period 1, firms purchase and (pay for) the services of inputs (labor, machinery and so forth) and use them to produce consumption goods and services that will be sold at the beginning of period 2, at which time all firms are disbanded. Firms finance their outlays for production in period 1 by issuing shares in their market values (= sale of output at the beginning of period 2) and these shares are investment assets held by consumers. It is the process by which period 1 market prices of such assets are determined.

The objective here is to analyse the nature of equilibrium in the capital market, and in particular on the measurement of the risks of assets and portfolios and the relationship between risk and equilibrium expected returns. The optimal consumption-investment decisions by individuals determine the risk structure of equilibrium expected returns. This analysis proceeds from partial equilibrium (consumption-investment) to capital market equilibrium—all the time, taking optimal production-investment decisions by firms and equilibrium in the markets for labor and current consumption goods as given.

Assumptions that all distribution of portfolio returns are normal and the consumers are risk averse imply that any expected utility maximising portfolio must be a member of $E(\tilde{R}_p)$, $\sigma(\tilde{R}_p)$, efficient set, where $E(\tilde{R}_p)$ is the expected return of the portfolio and $\sigma(\tilde{R}_p)$ is its standard deviation. An efficient portfolio is one that has maximum expected return for a given variance, or minimum variance for a given expected return. When a general equilibrium is reached at the beginning of period 1, the market value of consumer resources or his wealth w_i is determined and there is an optimal (that is, expected utility maximising) allocation of w_i between initial consumption c_1 and investment $(w_i - c_1)$ is some optimal portfolio of shares.

Since the model involves only risky assets, Sharpe has shown that the set of mean-deviation efficient portfolios form concave curve in mean-standard deviation space Further assumption that there are risk-free borrowing and lending opportunities available in the market and that all consumers can borrow or lend as much as they like at the risk-free rate R_f, the efficient set in the presence of risk-free borrowing and lending opportunities becomes straight line. Since the expectations and portfolio opportunities are homogenous throughout the market for all investors. Thus when equilibrium is attained all investors face efficient set. And efficient portfolio is now represented by portfolio m. The m is market portfolio, that is m consist of all assets in the market each entering the portfolio with weight equal to the ratio of its total market value to the total market value of all assets. In addition R_f must be such that net borrowing are zero, that is rate of R_f the total quantity of funds that people want to borrow is equal to the quantity that others want to lend.

Sharpe and Lintner thus making a number of assumptions extended Markowitz's mean variance framework to develop a relation for expected return, which can be written as[1]

$$E(R_i) = R_f + \beta_i ((E(R_m) - R_f) \quad \dots \quad \dots \quad \dots \quad \dots \quad (1)$$

where $E(R_i)$ is expected return on ith security, R_f is risk-free rate, $E(R_{m.})$ is expected return on market portfolio and β_i is the measure of risk or definition of market sensitivity parameter defined as cov(R_i, R_m)/var(R_m). Thus given that investors are risk averse, it seems intuitively sensible that high risk (high beta) stock should have higher expected return than low risk (low beta) stocks. In fact this is the just what the asset pricing model given by relation (1) implies. It says that in equilibrium an asset with zero systematic risk (β=0) will have expected return just equal to that on the riskless asset R_f, and expected return on all risky securities (β>0) will be higher by a risk premium which is directly proportional to their risk as measured by β.

Intuitively, in a rational and competitive market investors diversify all systematic risk away and thus price assets according to their systematic or non-diversifiable risk. Thus the model invalidates the traditional role of standard deviation as a measure of risk. This is a natural result of the rational expectations hypothesis (applied to asset markets) because if, on the contrary, investors also take into account diversifiable risks, then over time competition will force them out of the market. If, on the contrary, the CAPM does not hold, then the rationality of the asset's markets will have to be reconsidered.

In risk premium form CAPM Equation (1) can be written as

$$E(R_i) - R_f = \beta_i ((E(R_m) - R_f) \quad \dots \quad \dots \quad \dots \quad \dots \quad (2)$$

or $$E(r_i) = \beta_i E(r_m) \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (3)$$

[1]The derivation of Sharpe-Lintner CAPM is given in Appendix A.

where r_i is excess return on asset i and r_m is excess return on market portfolio over the risk-free rate. Equation (2) says that expected asst risk premium is equal to its β factor multiplied by the expected market risk premium.

Testing the CAPM theory relies on the assumption the ex-post distribution from which returns are drawn is ex-ante perceived by the investor. It follows from multivariate normality, that Equation (2) directly satisfies the Gauss-Markov regression assumptions. Therefore when CAPM is empirically tested in the literature it is usually written as following form,

$$r_i = \gamma_0 + \gamma_1 \beta_i + \varepsilon_i \quad \ldots \quad \ldots. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

$$E(\varepsilon_I) = 0 \quad \text{and} \quad \operatorname{cov}(r_m, \varepsilon_i)$$

In the Equation (4) an intercept term γ_0 is added, the term γ_1 is excess return of market over risk free rate and r_i is excess return on asset *i*. If γ_0=0 and γ_1>0, then CAPM holds.

The CAPM is a relationship between the ex-ante expected returns on the individual assets and the market portfolio. Such expected returns of course are not directly and objectively measurable. The usual procedure in such cases is to assume that the probability distribution generating the ex-post outcomes is stationary over time and then to substitute the sample average return for the ex-ante expectations.

Most tests of the asset pricing models have been performed by estimating the cross sectional relation between average return on assets, and their betas over some time interval and comparing the estimated relationship implied by CAPM. The time series estimation approach is also used in the literature. With the assumption that returns are *iid* and normally distributed the maximum likelihood estimation technique can be used to estimate the parameters γ_0 and γ_1.

(b) Capital Asset Pricing Model: Black Version

In the absence of riskless asset Black (1972) has suggested to use zero beta portfolio R_z that is cov(R_z, R_m) = 0, as a proxy for riskless asset In this case CAPM depends upon two factors; zero beta and non zero beta portfolios, and it is refereed as two factor CAPM, which may be represented as,[2]

$$E(R_i) = E(R_z) + \beta_i [E(R_m) - E(R_z)] \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

In excess return form

$$E(R_i) - E(R_z) = \beta_i [E(R_m) - E(R_z)] \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

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[2]Derivation of Black CAPM is given in Appendix B.

The zero-beta model specifies the equilibrium expected return on asset to be a function of market factor defined by the return on market portfolio R_m and a beta factor defined by the return on zero-beta portfolio-that is minimum variance portfolio which is uncorrelated with market portfolio. The zero-beta portfolio plays the role equivalent to risk free rate of return in Sharpe-Lintner model. The intercept term is zero implies that CAPM holds. Gibbons (1982), Stambaugh (1982) and Shanken (1985) have tested CAPM by first assuming that market model is true, that is the return as the ith asset is a linear function of a market portfolio proxy.

$$R_i = \alpha_i + \beta_i R_m + \eta_t \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

Black (1972) two-factor version requires the intercept term E (R_z) to be the same for all assets. Gibbons (1982) points out that the Black's two factor CAPM requires the constraint on the intercept of the market model

$$\alpha_i = E(R_z)\ (1\text{-}\beta_i)$$

for all the assets during the same time interval. When the above restriction is violated the CAPM must be rejected

Stambaugh (1982) has estimated the market model and using the Langrange multiplier test has found evidence in support of Black's version of CAPM. Gibbons (1982) has used a similar method as used by Stambaugh but employed likelihood ratio test (LRT), MacBeth (1975) has used Hotelling T^2 statistics to test the validity of CAPM.

(c) Capital Asset Pricing Model: Conditional Version

The traditional CAPM, which describes stock return solely on β measure, is based on the assumption that all market participants share identical subjective expectations of mean and variance of return distribution, and portfolio decision is exclusively based on these moments. But empirical evidence from literature suggests a deviation of the model from its formal theory. It has been observed that return distribution varies over time [Engle (1982) and Bollerslev (1986)]. In other words, the stock return distribution is time variant in nature and hence, the subjective expectation of moment differ from one period to another. This implies that the investor expectations of moments behave like random variables rather than constant as assumed in the traditional CAPM for stock returns.

The main proposition while taking care of time varying moments in CAPM is that, the investors still share identical subjective expectations of moments but these moments are conditional on the information at the time t. In symbols the conditional version of Sharpe-Lintner CAPM hereafter referred as conditional CAPM from Equation (1) can be written as

$$E(R_{it} \mid \psi_{t-1}) = E(R_{ft} \mid \psi_{t-1}) + \beta_{imt}[E(R_{mt} \mid \psi_{tt-1}) - E(R_{ft} \mid \psi_{t-1})] \quad \ldots \quad (8)$$

or in excess return form

$$E(R_{it} \mid \psi_{t-1}) - E(R_{ft} \mid \psi_{t-1}) = \beta_{imt}[E(R_{mt} \mid \psi_{t-1}) - E(R_{ft} \mid \psi_{t-1})] \quad \ldots \quad (9)$$

where E(R_{it}) is expected return on asset *i* on time *t*, R_{ft} return on riskless asset, ψ_{t-1} is the information set available at time *t*–1 and β_{imt} is the beta measure which is defined as $\beta_{imt} = \mathrm{cov}(R_{it}, R_{mt}!\psi_{t-1}) / \mathrm{var}(R_{mt}!\psi_{t-1})$. Equation (9) says that asset access return is proportion to conditional covariance of its asset return. As the return on riskless asset at the time t is known in advance at time t-1 and being included in ψ_{t-1} the conditional CAPM given in Equation (8) and (9) may be restated as

$$E(R_{it} \mid \psi_{t-1}) = R_{ft} + \beta_{imt}[E(R_{mt} \mid \psi_{t-1}) - R_{ft})] \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

Or excess return form

$$E(R_{it} \mid \psi_{t-1}) - R_{ft} = \beta_{imt}[E(R_{mt} \mid \psi_{t-1}) - R_{ft})] \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

The above CAPM form is conditional on information set ψ_{t-1} available at time *t*-1. Following Bodurtha and Mark (1991) it is plausible to express CAPM conditional on the given information set ψ_{t-1} in terms of its sub set say I_{t-1}. They have shown that if the CAPM holds in the sub set I_{t-1}, then it is also said to hold conditionally on ψ_{t-1}. In other words, the evidence that the CAPM conditional on I_{t-1} is not rejected implies acceptance of CAPM conditional on ψ_{t-1}. Following the proposition the CCAPM is specified as

$$E(R_{it} \mid I_{t-1}) - R_{ft} = \beta_{imt}[E(R_{mt} \mid I_{t-1}) - R_{ft}] \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

where

$$\beta_{imt} = \mathrm{cov}(R_{it}, R_{mt} \mid I_{t-1}) / \mathrm{var}(R_{mt} \mid I_{t-1}) \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

The test of CCAPM in Equation (12) becomes difficult due to the problem of obseving expected market return. To alleviate this problem, Bollerslev *et al.* (1988); Hall *et al.* (1989) and Ng (1991) suggest to assume market price of risk to be constant by defining as

$$\lambda = [(E(R_{mt} \mid I_{t-1}) - R_{ft}) / \mathrm{var}(R_{mt} \mid I_{t-1})] \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

where λ refers to market price risk. Hence expected return on the market portfolios may be represented as

$$E(R_{mt} \mid I_{t-1}) = R_{ft} + \lambda\, \mathrm{var}(R_{mt} \mid I_{t-1}) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

Equation (15) may be written as

$$R_{mt} = R_{ft} + \lambda\,\text{var}(R_{mt} \mid I_{t-1}) + u_{mt} \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \; (16)$$

where

$$u_{mt} = R_{mt} - R_{ft} + \lambda\,\text{var}(R_{mt} \mid I_{t-1}) \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \; (17)$$

Similarly Equation (12) may be rewritten as

$$R_{it} = R_{ft} + \lambda\,\text{cov}(R_{it}, R_{mt} ! I_{t-1}) + u_{it} \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \; (18)$$

where

$$u_{it} = R_{it} - E(R_{it} \mid I_{t-1}) \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \; (19)$$

Equations (16) and (18) represent return on market index and asset *i* respectively in regression form. In regression model given in Equation (18), a large shock in R_{it} is generally represented by a large deviation of R_{it} from $(R_{it} + \lambda\,\text{cov}(R_{it}, R_{mt} ! I_{t-1}))$ or equivalently a large positive or negative value of $u_{it.}$ Similarly, a large positive/negative deviation of u_{mt} reveals a large shock in R_{mt}. Further u_{it} and u_{mt} are orthogonal to information set I_{t-1}. Hence the conditional covariance between R_{it} and R_{mt} may be expressed as,

$$\text{cov}(R_{it}, R_{mt} \mid I_{t-1}) = E(u_{it}, u_{mt} \mid I_{t-1}) \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \; (20)$$

$$\text{var}(R_{mt} \mid I_{t-1}) = E(u_{mt}^2 \mid I_{t-1}) \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \; (21)$$

By incorporating Equation (20) and (21) the CCAPM in Equation (18) may be redefined as

$$R_{it} = R_{ft} + \lambda\,\text{cov}(u_{it}, u_{mt} \mid I_{t-1}) + u_{it} \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \; (22)$$

Equation (22) represents the cross-sectional relation between asset return i and its conditional covariance with market in terms of their errors. Hence the test of CAPM requires the functional specification of variance and covariance structure given in Equation (22).

In earlier research works the presence of time varying moments in return distribution has been in the form of clustering large shocks of dependent variable and thereby exhibiting a large positive or negative value of the error term [Mandelbrot (1963) and Fama (1965)]. A formal specification was ultimately proposed by Engle (1982) in the form of Autoregressive Conditional Hetroscedastic (ARCH) process. Some of latter studies have attempted to improve upon Engle's ARCH specification [Engle and Bollerslev (1986)]. The approaches which are helpful in specifying functional form of error term in the test of CCAPM include the approaches given by Engle and

Bollerslev (1986); Bollerslev *et al.* (1992) and Ng *et al.* (1992) in case of family of ARCH model.

In terms of error distribution, the Engle (1982), ARCH process may be represented as

$$r_{it} = a + br_{mt} + \varepsilon_t \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (23)$$

where r_{it} is excess return on asset i, r_{mt} is excess market return and ε_t is error term. The ARCH model characterises the random error term ε_t to be conditional on realised value of the set $\psi_t = (r_{t-j}, r_{mt-j} ....), j = 1,2,....n$. More specifically, the error ε_t is expected to follow the following assumptions

$$\varepsilon_t / \psi_{t-1} \sim N(0, \sigma_t^2) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (24)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + ... + \alpha_p \varepsilon_{t-p}^2 \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

Equation (24) states that the distribution of the current error term ε_t conditional on the given information set is normal with mean zero and variance, which is not a constant. Further Equation (25) states that the variance of the current error, conditional on the past error (ε_{t-j} j = 1,2,…n) is monotonically increasing function of its past error and hence heteroscedastic. Mandelbrot (1963) has observed that large (small) changes are tend to be followed by large (small) changes and its unconditional distribution has thick tails. As ARCH model characterises the error term ε_t conditional on information set, it can mimic the clustering of large shocks by exhibiting large (small) errors of either sign to be followed by large (small) error of either sign [Bera and Higgens (1995)]. Hence the application of ARCH appears to be a natural choice to express conditional variance given in Equation (25). The order of p in Equation (25) shows the period of shocks persistence in conditioning variance of current error, and conditional variance of pth order is denoted by ARCH (p).

Bollerslev (1986) has specified a generalisation of ARCH model referred as GARCH model, where

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + ... + \alpha_p \varepsilon_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + ... + \beta_q \sigma_{t-q}^2 \quad \ldots \quad \ldots \quad (26)$$

Equation (26) says that the conditional variance is function of past errors and past variances. The Equation (26) is referred as GARCH (p,q) process where p denotes the order of ε_t and q that of σ_t^2.

The implicit assumption of Engle ARCH and Bollerslev GARCH is that return distribution characterised with time variation only in variance. But the evidence on various studies have shown time variation in both mean and variance of return distribution [Domowitz and Hakkins (1985)]. Incorporating this idea Engle *et al*. (1987) has proposed the ARCH-M to account for time variation in both mean and variance. It may be represented as.

$$r_{it} = a + br_{mt} + f(\sigma_t^2) + \varepsilon_t \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (27)$$

where $\varepsilon_t / \psi_{t-1} \sim N(0, \sigma_t^2)$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \ldots.. + \alpha_p \varepsilon_{t-p}^2$$

The inclusion of $f(\sigma_t^2)$ is conditional variance function in equation (27) may be interpreted as a risk premium. If an asset is associated with higher risk, it is expected to yield a higher return. Hence the volatility of risk represented by variance attempted to explain the increase in the expected return due to increase in variance (risk) of the asset.

Bollerslev (1988) has formulated a model GARCH-M to account for time varying moments more efficiently; the model may be formulated as

$$r_{it} = a + br_{mt} + f(\sigma_t^2) + \varepsilon_t \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (28)$$

$$\varepsilon_t / \psi_{t-1} \sim N(0, \sigma_t^2) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \ldots.. + \alpha_p \varepsilon_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \ldots. + \beta_q \sigma_{t-q}^2 \quad \ldots \quad (30)$$

The test of ARCH or any other variant like GARCH or GARCH-M is carried out by a simultaneous estimation of parameters in mean and variance. For instance the test of GARCH-M requires a simultaneous estimation of parameters in Equation (28), (29) and (30) respectively. As the error variance is expressed in non-linear form, a non-linear optimisation procedure is required for estimation. Ng (1991) and Bollerslev, Engle and Woldridge (1988) used ARCH-M model and maximum likelihood as estimation procedure. Harvey (1989) and Bodurtha and Mark (1991) generalised method of moments (GMM) as estimation technique.

(d) Arbitrage Pricing Theory

The arbitrage pricing theory (APT) is originally proposed by Ross (1976) and latter extended by Huberman (1982), Chamberlain and Rothschild (1983), Chen and Ingersoll (1983) Connor (1984), Chen (1983), Connor and Korajczky (1988) and Lehmann and Modest (1988), and numerous other researchers. The APT has recently attracted considerable attention as a testable alternative to capital asset pricing model of Sharpe-Lintner and Black. The APT states that, under certain assumptions, the single period expected return on any risky asset is approximately linearly related to its associated factor loadings (i.e., systematic risks) as shown below,

$$\tilde{R}_i = E(\tilde{R}_i) + b_{i1}\,\tilde{F}_1 + \ldots + b_{ik}\,\tilde{F}_k + \tilde{\varepsilon}_k, \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

where $\tilde{R}_i$ is the random rate of return on the ith asset, $E(\tilde{R}_i)$ is the expected rate of return on the ith asset, b_{ik} is the sensitivity of the ith asset's returns to the kth factor, $\tilde{F}_k$ is the mean zero kth factor common to the returns of all assets under considerations, $\tilde{\varepsilon}_k$ is random zero mean noise term for the ith asset.

The APT is derived under the usual assumptions of perfectly competitive and frictionless capital markets. Furthermore, individuals are assumed to have homogeneous beliefs that the random returns for the set of assets being considered are governed by the linear k-factor model given in Equation (31). The theory requires that the number of assets under consideration, n, be much larger than the number of factors, k, and that the noise term, $\tilde{\varepsilon}_i$ be the unsystematic risk component for the ith asset. It must be independent of all factors and all error terms for other assets.

The basic idea of APT is that in equilibrium all portfolios that can be selected from among the set of assets under consideration and that satisfy the conditions of (a) using no wealth and (b) having no risk must earn no return on average. These portfolios are called arbitrage portfolios. To see how they can be constructed, let w_i be the wealth invested in the ith asset as a percentage of an individual's total invested wealth. To form an arbitrage portfolio that requires no change in wealth, the usual course of action would be to sell some assets and use the proceeds to buy others. Thus the zero change in wealth is written as

$$\sum_{i=1}^{n} w_i = 0. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (32)$$

If there are n assets in the arbitrage portfolio, then the additional portfolio return gained is

$$\tilde{R}_p = \sum_{i=1}^{n} w_i\,\tilde{R}_i$$

$$= \sum_{i} w_i\,E(\tilde{R}_i) + \sum_{i=1}^{n} w_i\,b_{i1}\,\tilde{F}_1 + \ldots.. + \sum_{i} w_i\,b_{ik}\,\tilde{F}_k + \sum_{i} w_i\,\tilde{\varepsilon}_i \quad \ldots (33)$$

To obtain a riskless arbitrage portfolio it is necessary to eliminate both diversifiable (i.e., unsystematic or idiosyncratic) and undiversifiable (i.e., systematic) risks. This can be done by meeting three conditions: (1) selecting percentage changes in investment ratios w_i, that are small, (2) diversifying across a large number of assets, and (3) choosing changes w_i, so that for each factor, k, the weighted sum of the systematic risk components, b_k, is zero. These conditions can be written as follows,

$$w_i \approx 1/n, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (34a)$$

$$n \text{ chosen to be a large number,} \quad \dots \quad \dots \quad \dots \quad \dots \quad (34b)$$

$$\sum_i w_i \, b_{ik} = 0 \text{ for each factor.} \quad \dots \quad \dots \quad \dots \quad \dots \quad (34c)$$

Because the error terms, $\tilde{\varepsilon}_i$ are independent, the law of large numbers guarantees that a weighted average of many of them will approach to zero in the limit as n becomes large. In other words, costless diversification eliminates the last term i.e., idiosyncratic risk in Equation (31). Thus we are left with

$$\tilde{R}_p = \sum_i w_i \, E(\tilde{R}_i) + \sum_i w_i \, b_{i1} \, \tilde{F}_1 + \dots\dots + \sum_i w_i \, b_{ik} \, \tilde{F}_k \quad \dots \quad \dots \quad (35)$$

Since we have chosen the weighted average of the systematic risk components for each factor to be equal to zero ($\sum_i w_i \, b_{ik} = 0$), this eliminates all systematic risk. This can be considered as selecting an arbitrage portfolio with zero beta in each factor. Consequently, the return on the arbitrage portfolio becomes a constant because of the choice of weights has eliminated all uncertainty. Therefore Equation (33) can be written as,

$$R_p = \sum_i w_i \, E(\tilde{R}_i) \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (36)$$

Since the arbitrage portfolio is so constructed, that it has no risk and requires no new wealth. If the return on the arbitrage portfolio were not zero, then it would be possible to achieve an infinite rate of return with no capital requirements and no risk. Such an opportunity is clearly impossible if the market is to be in equilibrium. In fact, if the individual investor is in equilibrium, then the return on any and all arbitrage portfolios must be zero. This can be expressed as,

$$R_p = \sum_i w_i \, E(\tilde{R}_i) = 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (37)$$

From no wealth constraint represented by Equation (32), any orthogonal vector to this constraint vector can be formed as given below

$$\left(\sum_i w\right). e = 0, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (38)$$

and to each of the coefficient vectors from Equation (34c), i.e.,

$$\sum_i w_i \, b_{ik} = 0 \qquad \text{for each } k,$$

and must also be orthogonal to the vector of expected returns, Equation (37), i.e.,

$$\sum_i w_i \, E(\tilde{R}_i) = 0.$$

Thus the expected return vector can be written as a linear combination of the constant vector and the coefficient vectors. That is, there must exist a set of k + 1 coefficients, $\lambda_o + \lambda_{1,}.....,\lambda_k$ such that

$$E(\tilde{R}_i) = \lambda_o + \lambda_1 \, b_{i1} + \ldots. + \lambda_k \, b_{ik} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (39)$$

Since b_{ik} are the sensitivities of the returns on the *i*th security to the *k*th factor. If there is a riskless asset with a riskless rate of return, R_f , then b_{ok} = 0 and $R_f = \lambda_o$. Hence Equation (39) can be rewritten in excess returns form as follows,

$$E(R_i) - R_f = \lambda_1 \, b_{i1} + \ldots + \lambda_k \, b_{ik} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (40)$$

The arbitrage pricing relationship (40) says that the arbitrage pricing relationship is linear and λ represents the risk premium (i.e., the price of risk), in equilibrium, for the *k*th factor. Now rewrite Equation (40) as

$$E(R_i) \text{ - } R_f + [\, \bar{\delta}_k - R_f \,] \, b_{ik} \, , \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (41)$$

where $\bar{\delta}_k$ is the expected return on a portfolio with unit sensitivity to the *k*th factor and zero sensitivity to all other factors. Therefore the risk premium, λ_k , is equal to the difference between the expectation of a portfolio that has unit response to the *k*th factor and zero response to the other factors and the risk rate, R_f .

Thus the APT model is represented by following equation,

$$E(R_i) \text{ - } R_f = [\, \bar{\delta}_k - R_f \,] \, b_{i1} + \ldots. + [\, \bar{\delta}_k - R_f \,] \, b_{ik} \, , \quad \ldots \quad \ldots \quad (42)$$

The Equation (42) represents a linear regression equation and coefficients, b_{ik} , are defined in exactly the same way as beta in the capital asset pricing model represented by Equation (4)

Chamberlain and Rothschild (1983) and Ingersoll (1983) have extended Ross (1976) result by showing that Equation (42) holds even for an approximate factor structure. In an approximate factor structure, it is assumed that the $\tilde{\varepsilon}_k$ in Equation (31) are correlated with each other and that the eigenvalues of the covariance matrix of $\tilde{\varepsilon}_k$ are uniformly bounded from above by some finite number. The notion of an approximate factor seems to be a significantly weaker restriction on the return generating process than the Ross strict structure. However, Grinblatt and Titman (1983) illustrates that ant finite economy satisfying the approximate factor structure may be transformed into another finite economy satisfying the Ross strict factor structure in a manner that does not alter the characteristics of investors' portfolios. In other words, a strict factor structure is equivalent to an approximate factor structure in an infinite economy.

Connor (1982) has employed a competitive equilibrium assumption to show that the elimination of infinite security assumption does not change the pricing relation if the market portfolio is well diversified in a given factor structure. A competitive equilibrium consist of set of portfolios such that all portfolios are budget constraint optimal for every investor and security supply equal to security demand. In a competitive equilibrium, there exists an exactly linear pricing relation in such asset factors betas or sensitivities that Equation (42) holds exactly. Chen and Ingersoll (1983) have reached the same conclusion provided that a well diversified portfolio exists in a given factor structure and this portfolio is the optimal portfolio for at least one utility maximising investor. More specifically the pricing relation of the APT, given either of these diversified portfolio assumptions, is exact in the finite economy.

A major problem in testing Arbitrage Pricing Theory is that the pervasive factors affecting asset returns are unobservable. The conventional factor extraction techniques are maximum likelihood factor analysis and principle component approach. Mostly factor analysis to measure these common factors has been used [Chen (1983); Roll and Ross (1980); Reinganum (1981); Lehmann and Modest (1988)]. While Connor and Korajczyk (1988) have used the asymptotic principal component technique to estimate the pervasive factors influencing asset returns and to test the restrictions imposed by static and intertemporal version of APT on a multivariate regression model. The factor extraction analysis is only a statistical tool to uncover the pervasive forces (factors) in the economy by examining how asset return covary together.

In using maximum likelihood procedure, if one knows the factor loadings for say k portfolio, then one can compute the k factor loadings for all securities [Chen (1983)]. We can use factor analysis only on one group of securities or portfolios and the factor loadings of all securities will correspond to the same common factor. Since b_{ik} are not observable, we need to construct a proxy for the factor loadings. In factor analysis we can use estimated b as proxy, then run a cross-sectional regression of R_{it} on b_{ik}. We can use autoregressive approach as well and derive proxy from the return generating process. The intuition behind this is that historical excess returns are useful in explaining current cross sectional returns because they span the same return space as b_{ik}, and thus can be used as proxies for systematic risks. The substitution of excess return for unobservable b_{ik} is similar in spirit to the technique of substituting mimicking factors portfolios return for unobservable factors used by Jobson (1982). After identifying the factor, we use the estimated factor loadings to explain the cross sectional variation of individual estimated expected returns and to measure the size and statistical significance of the estimated risk premia associated with each factor.

2. REVIEW OF EMPIRICAL LITERATURE AND ITS IMPLICATIONS

The capital asset pricing models have been subjected to extensive empirical testing in the past 30 years. The early extensive studies of Sharpe-Lintner-Black (SLB) model are Black, Jensen and Scholes (1972); Blume and Friend (1973); Fama and MacBeth (1973); Basu (1977); Reinganum (1981); Banz (1981); Gibbons (1982); Stambaugh (1982) and Shanken (1985). However, in general the results have offered very little support of the CAPM model. These studies have suggested that a significant positive relation existed between realised return and systematic risk as measured by β, and relation between risk and return appeared to be linear. But the special prediction of Sharpe-Lintner version of the model, the portfolio uncorrelated with market have expected return equal to risk free rate of interest, have not done well, and the evidence have suggested that the average return on zero-beta portfolios are higher than risk free rate.

Most of early test of CAPM have employed the methodology of first estimating betas using time series regression and then running a cross section of regression using the estimated betas as explanatory variables to test the hypothesis implied by the CAPM.

The first tests of CAPM on individual stock in the excess return form have been conducted by Lintner (1965) and Douglas (1968). They have found that the intercept has value much larger than R_f, the coefficient of beta is statistically significant but has a lower value and residual risk has effect on security returns. Their results seem to be a contradiction to the CAPM model. But both the Daglas and Lintner studies appear to suffer from various statistical weaknesses that might explain their anomalies results. The measurement error has incurred in estimating individual stock betas, the fact that estimated betas and unsystematic risk are highly correlated and also due to skewness present in the distribution of observed stock returns. Thus Lintner's results have seemed to be in contradiction to the CAPM.

As regards the test of CAPM on portfolios, one classic test was performed by Fama and MacBeth (1973). They have combined the time series and cross sectional steps to investigate whether the risk premia of the factors in the second pass regression are non-zero. Forming twenty portfolios of assets, they have estimated beta from time series regression methodology, they then performed a cross sectional regression for each month over the period 1935–68 in the second pass regression. Their results have shown that the coefficient of beta was statistically insignificant and its value has remained small for many sub-periods. But in contrast to Lintner, they have found residual risk has no effect on security returns. However, their intercept is much greater than risk free rate and the results indicate that CAPM might not hold.

Black, Jensen and Scholes (1972) have tested CAPM by using time series regression analysis. The results have shown that the intercept term is different from zero and in fact is time varying. They have found when $\beta \succ 1$ the intercept is negative and that it is positive when $\beta \prec 1$. Thus the findings of Black et al violate the CAPM.

Stambaugh (1982) has employed slightly different methodology. He has estimated the market model and using Lagrange multiplier (LM) test has found evidence in support of Black's version of CAPM, but has not conformed the validity of Sharpe-Lintner CAPM. Gibbons (1982) has used a similar method as the one used by Stambaugh but instead of LM test he has used maximum likelihood ratio test and reject the both standard and zero beta CAPM.

The test of market efficiency jointly with equilibrium asset pricing model has been focus of many studies and excellent review of this literature is provided by Fama (1970, 1991). Market efficiency hypothesis is that security prices reflect fully all available information. The equilibrium asset pricing models generally imply that the market portfolio is ex-ante mean variance efficient in the sense of Markowitz (1959). For both the CAPM and APT to be true, the asset prices must be efficient price, but the reverse is not necessary. In many situations in rejecting CAPM or APT, it is difficult to tell whether the risk-return relation represented by these models is incorrect or market is inefficient.

In a well-known paper Roll (1977) has made a serious methodological criticism of the empirical tests of Sharpe-Lintner-Black (SLB) model. He has argued that the early tests were not much evidence for the validity of SLB model because the proxies used for the market portfolio do not come close to the portfolio of invested wealth called by the model. He has pointed out that the test performed by using any other portfolio other than the true market portfolio are not test of CAPM but are tests of whether the proxy portfolio is efficient or not. But Stambaught (1982) has shown that tests of the SLB model are not sensitive to the proxy used for the market and have suggested that Roll's criticism is too strong. He has expanded the type of investments included in his proxy from stocks listed on New York Stock Exchange to corporate and government bonds to real estate to durable goods such as house furnishing and automobiles. His results have indicated that the nature of conclusion is not materially effected as one expands the composition of the proxy for the market portfolio. But this issue can never be entirely resolved.

Some of the most important findings of Sharpe-Lintner-Black model are anomalies. The empirical attack on this model has begun with the studies that have identified variables other than market β to explain cross-section of expected returns. Basu (1977) have showed that earning-to-price ratio have marginal explanatory power after controlling for β, expected returns are positively related to E/P. Banz (1981) has found that a stock size (price times share) could help explain expected returns, given these market β, expected returns on small stocks are too high and expected returns on large stocks are too low. Bhandari (1988) has explored that leverage is positively related to expected stock returns, Fama and French (1992) have found that higher book-to-market ratios are associated with higher expected return, in their tests that also include market β.

These anomalies are now stylised facts to be explained by multifactor asset pricing models of Merton (1973) and Ross (1976). For example Ball (1978) have argued that E/P is a catch-all proxy for omitted factors in asset pricing tests and one can expect it to have explanatory power when asset pricing follow a multifactor model and all relevant factors are not included. Chan and Chen (1991) have argued that size effect is due to the fact that small stocks include many martingale or depressed firms whose performance is sensitive to business conditions. Fama and French (1992) have shown that since leverage and book-to-market equity are also largely driven by market value of equity, they also may proxy for risk factors; in return that are related to market judgments about the relative prospects of firms. One can expect when asset pricing follow a multifactor models and all relevant factors included in the asset pricing tests to explain these anomalies. There are some other research works, which have shown that there is indeed spill over effect among Sharpe-Lintner anomalies. Basu (1983) have found that size and E/P are related; Fama and French (1992) have found that size and book to market equity are related and again leverage and book in market equity are highly correlated.

These multifactor asset pricing model generalise the result of SLB model. In these models, the return generating models involve multiple factors and the cross section of expected returns is explained by the cross section of factor loadings or sensitivities. One approach suggested by Ross (1976) arbitrage pricing theory (APT) uses factor analysis to extract the common factors and then tests whether expected returns are explained by the cross section of the loading of asset returns on the factor [Roll and Ross (1980); Chen (1983); Lehmann and Modest (1988)] have tested this approach in detail. The factor analysis approach to test of the APT leads to unreasonable conflict about the number of common factors and what these factors are. The factor analysis approach is limited, but it confirms that there is more than one common factor in explaining expected returns.

The alternate approach in Chen, Roll and Ross (1986) is to look for economic variables that are correlated with stock returns and then to test whether the loading of these economic factors describe the cross section of expected returns. This approach thus gives insight about how the factors relate to uncertainties about consumption and portfolio opportunities that are of concern to investor. They have examined a range of business condition variables that might be related to return because they are related to shocks to expected future cash flows or discount rate. The most powerful variables are the growth rate of industrial production and difference between the return on long term low grade corporate bonds and long terms government bonds The unexpected inflation rate and the difference between the return on long and short government returns are found to be less significant. Merton (1973) has constructed a generalised intertemporal asset pricing model in which factors other than market uncertainty are priced. In his model individuals are solving their lifetime consumption decision in a multi-period setting. He has shown that return on assets depend not only on the covariance of asset with the market but also with the covariance with changes in investment opportunity set and thus can be interpreted as another form of APT. Fama and French (1992) in their influential paper have shown that that two variables size and book-to-market-equity combine to capture the cross-sectional variation in average stock return associated with market beta, size, leverage, book-to-market and earning-to-price ratio. But the problem of simplicity of Chen, Roll and Ross approach can be a trap, and measured relation between returns economic factors may be spurious as the result of a particular sample chosen and therefore robustness checks are needed.

Using SLB model some studies have been done to evaluate investment performance of mutual fund, pension funds and endowment funds. Jensen (1968,1969), Chang and Lewellen (1984), Ippolito (1989) are important in this area of mutual fund industry. Investment performance of pension plans were studied by Beefower and Bergstrom (1977) and Ippolito and Turner (1987). The evidence suggest that one-factor Sharpe-Lintner model has many problems in explaining cross section of expected stock return and multifactor model seems to do a better job in evaluating investment performance. For example the three-factor performance evaluation method of Elton, Gruber, Das and Hklarka (1991) has given more insight in this issue.

After the event study of stock splits by Fama, Fishers, Jensen and Roll (1969), the event study has become important part of financial economics. When a stock price response to an event and returns are abnormal returns, how it affects the risk—return trade off is subject of event studies [Brown and Warner (1985)]. When an event is dated precisely and event has a large effect on prices, the way one abstracts from expected return to measure abnormal daily returns is important consideration along with the speed of adjustment of prices to information that is efficiency consideration. The unexpected changes in dividends on average associated with stock price changes and studied by Charest (1978). Millor and Scholes (1972). They have shown that either dividend policy is irrelevant or are bad news. Other event studies are on new issues of common stocks are bad news for stock prices [Asquith and Mullins (1986)] or good news [Myers and Majlufe (1984)]. Like financing decisions, corporate control transaction has been examined by use of these equilibrium models in event studies literature. One such issue is merger and tender offers on average produce large gains for the share holders of largest firms [Mandklker (1974) and Bradley (1980)].

Now as regards the empirical testing of selected stock exchanges, Green (1990) have tested CAPM on UK private sectors data and found that SLB model do not hold. But Sauer and Murphy (1992) have investigated this model in German stock market data and conformed CAPM as the best model describing stock returns. Another contradictory evidence has been found by Hawawini (1993) in equity markets in Belgium, Canada, France, Japan, Spain, UK and USA. The other studies, which tested CAPM for emerging markets are Lau *et al.* (1975) for Tokyo Stock Exchange, Sareewiwathana and Malone (1985) for Thailand stock exchange, Bark (1991) for Korean Stock Exchange and Gupta and Sehgal (1993) for Indian stock Exchange. Badar (1997) has estimated CAPM for Pakistan.

3. CRITICAL ANALYSIS OF THE REVIEW

The asset-pricing model has been subject of several academic papers; it is still exposed to theatrical and empirical criticism.

For example Miller and Scholes (1972) have discussed the statistical problem inherent in the empirical studies of CAPM. By using historical data they have found that R_f and R_m are negatively correlated, this would lead to an upward bias in intercept and slope would to biased downwards. However if R_f varies with time and correlated with R_{mt}, then we inevitably encountered the problem of omitted variables bias and thus the estimated betas will be biased. This is in fact what many studies have found, and thus the fact that these studies reject the CAPM does not imply that it does not hold.

Another factor that may bias intercept upward and slope downward is presence of hetroscedaticity. In addition biases may encounter in two-stage regression used by these studies, because estimated betas are used as variables in the second pass regression. Thus any error in the first stage is carried to the second stage.

Another possible problem in many early tests of CAPM have arisen due to it being a single period model. Most tests have used time series regression, which is appropriate, if the risk premia and betas are stationary, which is unlikely to be true.

Roll (1977) has shown that there has been no single unambiguous test of the CAPM. He pointed out that the test performed by using any portfolio other than the true market portfolio are not test of CAPM, but are tests of whether the proxy portfolio is efficient or not. Intuitively market portfolio includes all the risky assets including human capital while the proxy just contains the subset of all assets.

Black, Jensen and Scholes (1972) have not even mentioned the possible efficiency of market portfolio and conclude that the relationship between expected return and beta is not linear. This conclusion is enough to prove that the proxy used does not lie on the sample efficient frontier. Fama and MacBeth (1973) in their study have used the Fisher Arithmetic Index as equally weighted portfolio of all stocks in New York Stock Exchange as their proxy. This proxy is not even close to value-weighted portfolio and should not have used as market proxy. Thus the conclusion of Fama and MacBeth are also not immune to suspicion.

Furthermore, Roll has shown that the situation is aggravated by the fact that both the Sharpe-Lintner CAPM and Black version of CAPM are liable to type II error, i.e., likely to be rejected when they are true. This is true even if the proxy is highly correlated with true market portfolio. Thus the efficiency or inefficiency of the proxy does not imply anything about the efficiency of the true market portfolio.

The measurement error in testing CAMP may explain the observed size effect, as the betas for small firms are too low. If this is true the CAPM will give a smaller expected returns for small stocks and there will be measurement errors associated with beta. Christie and Hertzel (1981) have pointed out that those firms, which become small also become riskier but since beta is measured using historical returns, this does not capture this increased risk. Further Reinganum (1981) and Roll (1981) have shown that beta estimated for small firms will be biased downwards as they trade less frequently than do the larger firms. Lo and MacKinlay (1990) have argued the biases relating to data snooping may explain the observed deviation from the model. The firms that are not performing well are excluded. And since the falling stocks have a lower return and high book-to-market ratio, thus the included high book-to-market firms will be biased upward.

Korthari, Shanken and Sloan (1995) have argued this bias may explain the result found by Fama and French (1992).

Roll and Ross (1994) in their recent paper have pointed out again that a positive and exact cross-sectional relation between return and beta must hold if the market index used is mean-variance efficient. If such a relationship were not found than this would suggest that the proxy used is ex-ante inefficient. They have further stated that given that direct test have rejected the mean variance efficiency for many market proxies e.g., Shanken (1985) and others. It is not surprising that empirical studies have found that the role of other variables in explaining cross-sectional return is significant. However what is surprising is the fact that some studies [e.g., Fama and French (1992)] have shown that mean return-beta relationship is virtually zero.

One possible interpretation of the findings of the above section is that the factors found to be significant in the above studies may actually be correlated with the true market portfolio.

The choice of econometric technique is also important in this regard. Roll and Ross have shown that depending upon econometric technique used, one can get a range of different results with the same data. In particular, they have proposed that the use of GLS instead of OLS always produce a positive cross sectional relationship between expected return and betas. This is true regardless of the efficiency of the proxy as long as the return on the proxy is greater than the return on the minimum variance portfolio. Kandell and Stambaugh (1995) also advocated the use of GLS as they have shown that by using GLS, R^2 increases as the proxy lies closer to the efficient frontier and thus GLS can mitigate the extreme sensitivity of cross –sectional results. Amibul, Christensen and Mendelson (1992) by using GLS and by replicating Fama and French (1992) tests have found that in contrast to the results of Fama and French, beta significantly affects expected returns. However the problem with GLS is that the true parameters are unknown and hence the true covariance matrix of returns is also unknown. Further, since the use of GLS in almost very proxy producing a positive cross sectional relation between mean returns and betas, hence unless other tests of efficiency are carried out, the results are by themselves of little significance.

There are serious problems in empirically testing APT as well. Dhrymes, Friend and Gultekin (1984) have provided evidence that the number of common factors in test increases as the number of assets in sample increases or length of time period sampled increases. But Roll and Ross (1994) have responded that this would be expected. As additional securities or returns are collected, additional common factors might emerge. For example as sample size increases, firms from a number of new industries might be included that share a common factor. Roll and Ross have pointed out that it is the number of priced factor which are important not the total number of factors. Shanken (1992) has also criticised the testing of APT. He has argued that by altering the portfolios construction changes risk premia and the returns that are examined on securities can mask or exacerbate the underlying factor risks in the economy. But this problem is less sever in individual stocks. In case of portfolios even the firms are not constantly changing the nature of their assets portfolio, as in the case of mutual fund. The major criticism is that APT is silent regarding the particular systematic factors effecting a security risk and return. Investors must fend for themselves in determining these factors.

Underlying CAPM and APT the assumption is that the return generating process is stationary. But researchers have found evidence that the expected market risk premium is positively related to predicted volatility of stock returns [French, Schwert and Stambaugh (1987)].

Thus inspire of a number of anomalies in hand, the CAPM has done the job as expected of a good model. In rejecting it, our understanding of asset pricing has enhanced. These anomalies are now stylised facts to be explained by other asset pricing models such as multifactor asset pricing models of Merton (1973) and Ross (1976). These models are rich and more flexible than their competitor. Based on existing evidence, they have shown some promise to fill the empirical void left by rejecting the CAPM.

The potential usefulness of CAPM for practical investment and portfolio analysis has received increasing attention in the past thirty years in the professional financial community. It has given a summary measure of risk, market beta, interpreted as market sensitivity. Indeed, inspire of evidence against CAPM, market professional and academics still think about risk in terms of market beta. And like academics, practitioners retain the market line (from risk free rate through the market portfolio) of the Sharpe-Lintner model as a representation of the trade-off of expected return for risk available from passive portfolios. The popularity of CAPM is due to its potential testability. If empirically true, it has wide ranging implications for problem in capital budgeting, cost benefit analysis, portfolio selection and other economic problem requiring knowledge of relation between risk and return such as evaluation of investment performance and event studies and development of investment management strategies.

The inception of APT has provided has provided researcher and practitioners with an intuitive and flexible framework through which to address important investment management issues. One advantage is that APT operates under relatively weaker assumptions. Further because of its emphasis on multiple source of systematic risk, APT has attracted considerable interest as a tool for better explaining investment results and more efficiently controlling portfolio risks. The number of institutional investors actually using APT is small. The most prominent organisation is Roll and Ross Asset Management Corporation.

4. CONCLUSION

Considering the above analysis, it is not easy to give unambiguous conclusion. On the one hand, there is strong empirical evidence invalidating the capital asset pricing model and on the other hand it is clear that empirical findings are not themselves sufficient to discard the model. Indeed, as noted by many researcher including Fama and French in their articlr,'The CAPM is wanted, dead or alive', the empirical tests have been undermined by inability to observe the true market portfolio. In effect the estimated CAPM based on the proxy market index can be rejected, nevertheless it is virtually impossible to reject the theoretical CAPM.

More than a modest level of disappointment with the CAPM is evident by number of related but different theories, for example, Hakanson (1971); Merton (1973); Ball (1978); Ross (1976); Reinganum (1981), and by questioning of CAPM's validity, as a scientific theory, e.g., Roll (1977, 1994). Nonetheless, the CAPM remains a central place in the thoughts of finance practitioners such as portfolio managers, investment advisors and security analysts. But there is a good reason for its durability, the fact that it explains return common variability in terms of single factor, which generates return for each individual asset, via some linear functional relationship. The elegant derivation of CAPM is based on first principle of utility theory. But the attractiveness of the CAPM is due to its potential testability.

The important point to emphasise is that the Sharpe-Lintner-Black CAPM, conditional CAPM, consumption CAPM and multifactor model are not mutually exclusive. Following Constantinides (1989), one can view the models as different ways to formulise the asset pricing implications of common general assumptions about tastes (risk aversion) and portfolio opportunities (multivariate normality). Thus as long as major prediction of the models about the cross section of expected returns have been some empirical content, as long as we keep the empirical short comings of the models in mind, we have some freedom to lean on one model or another, to suit the purpose in hand.

Appendices

APPENDIX A

For an individual investor the relationship between the risk of an asset and its expected return is implied by the fact the investor's optimal portfolio is efficient. Thus if he chooses portfolio *m*, the fact that m is efficient means that the weights x_{im}, i=1…..N, maximises expected portfolio return.

$$\text{Maximise } E(\tilde{R}_m) = \sum_{i=1}^{n} x_{im} E(\tilde{R}_i) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Subject to the constraints

$$\sigma(\tilde{R}_p) = \sigma(\tilde{R}_m) \quad \text{and} \quad \sum_{i=1}^{N} x_{im} = 1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

The lagrangian methods show the solution is

$$E(\tilde{R}_j) - E(\tilde{R}_i) = S_m \left[\frac{\partial \sigma(\tilde{R}_m)}{\partial x_{jm}} - \frac{\partial \sigma(\tilde{R}_m)}{\partial x_{im}} \right] \quad i.j{=}1,2,\ldots\ldots n \quad \ldots \quad (3)$$

where $\partial\sigma(\tilde{R}_m)/\partial(x_{jm})$ is $\partial\sigma(\tilde{R}_p)/\partial(x_{jp})$ evaluated at the optimal value $x_{ip} = x_{im}, i = 1.2......N$. S_m is the Lagrange multiplier is shadow price of the constraint provides the rate of change of maximum of $E(\tilde{R}_p)$ for small changes in the allowed level of $\sigma(\tilde{R}_p))$ in the neighborhood of $\sigma(\tilde{R}_m)$. Equation (3) explains how the value of *x*'*s*, the proportion invested in individual asset must be chosen in order to obtain the efficient portfolio with dispersion $\sigma(\tilde{R}_m)$.

Now to develop risk-return relation from Equation (3), since this expression holds between assets and the efficient portfolio m as well as between individual assets themselves, therefore premultiply both sides by x_{im} and sum over i and equation becomes

$$E(\tilde{R}_j) - E(\tilde{R}_m) = S_m \left[\frac{\partial \sigma(\tilde{R}_m)}{\partial x_{jm}} - \sum_{i=1}^{N} x_{im} \frac{\partial \sigma(\tilde{R}_m)}{\partial x_{im}} \right] \quad \ldots \quad \ldots \quad (4)$$

$$\text{or} \quad E(\tilde{R}_j) - E(\tilde{R}_m) = S_m \left[\frac{\partial \sigma(\tilde{R}_m)}{\partial x_{jm}} - \sigma(\tilde{R}_m) \right] \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

It implies that, to form the efficient portfolio with dispersion of $\sigma(\tilde{R}_m)$, the proportion x_{im} invested in the individual asset must be such that the difference between the expected return on an asset and expected return on the portfolio is proportional to difference between the marginal effect of the asset on $\sigma(\tilde{R}_m)$. The Equation (5) can be interpreted as the relationship between expected return and risk for an individual asset, measured relative to efficient portfolio *m*. That is he difference between expected return on an asset and on the portfolio is proportion to the difference between the risk of an asset and the risk of the portfolio, so that the expected return on an asset is always a linear function of the risk.

Now we have concave curve representing efficient set in the $E(\tilde{R}), \sigma(\tilde{R})$ plane. Further assumption that there are risk-free borrowing and lending opportunities available in the market and that all consumers can borrow or lend as much as they like at the risk-free rate R_f. The efficient set in the presence of risk-free borrowing and lending opportunities becomes straight line. In this case, when market equilibrium is attained all consumers face the same efficient set *m* and expected return risk relationship is derived for any given efficient portfolio will be relevant for all investors who chose that portfolio.

Since the market portfolio m is efficient equation and riskless borrowing and lending is available, Equation (5) can be written as

$$E(\tilde{R}_i) - E(\tilde{R}_m) = S_m\left[\frac{\partial \sigma(\tilde{R}_m)}{\partial x_{im}} - \sigma(\tilde{R}_m)\right] \qquad \ldots \qquad \ldots \qquad \ldots \quad (6)$$

Where $S_m = [E(\tilde{R}_m) - R_f]/\sigma(\tilde{R}_m)$

and let $\operatorname{cov}(\tilde{R}_i, \tilde{R}_j)$ be the covariance between one period returns on the share of assets *i* and *j*, thus

$$\sigma(\tilde{R}_m) = \left[\sum_{j=1}^{N}\sum_{k=1}^{N} x_{jm} x_{km} \operatorname{cov}(\tilde{R}_j, \tilde{R}_k)\right]^{1/2}$$

and $$\frac{\partial \sigma(\tilde{R}_m)}{\partial x_{jm}} = \frac{\sum_{k=1}^{N} x_{im} \operatorname{cov}(\tilde{R}_j, \tilde{R}_k)}{\sigma(\tilde{R}_m)} = \frac{\operatorname{cov}(\tilde{R}_j, \tilde{R}_m)}{\sigma(\tilde{R}_m)}$$

so that Equation (6) becomes

$$E(\tilde{R}_i) = R_f + \left[\frac{E(\tilde{R}_m) - R_f}{\sigma(\tilde{R}_m)}\right] \frac{\operatorname{cov}(\tilde{R}_j, \tilde{R}_m)}{\sigma(\tilde{R}_m)} \qquad \ldots \qquad \ldots \qquad \ldots(7)$$

Now rewriting the Equation (7)

$$E(\tilde{R}_i) = R_f + \beta_i [E(\tilde{R}_m) - R_{f\,j}] \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \quad (8)$$

where $$\beta_i = \frac{\operatorname{cov}(\tilde{R}_i, \tilde{R}_m)}{\sigma^2(\tilde{R}_m)} = \frac{\sum_{j=1}^{N} x_i \sigma_{ij}}{\sigma^2(\tilde{R}_m)} = \frac{\operatorname{cov}(\tilde{R}_i, \tilde{R}_m)/\sigma(\tilde{R}_m)}{\sigma(\tilde{R}_m)}$$

The parameter βi can be interpreted as risk of asset i in portfolio m relative to $\sigma(\tilde{R}_m)$ the total risk of m. The Equation (8) is the main result of the Sharpe-Lintner CAPM, and this relation holds both for assets and portfolios. The Equation (8) says that expected return on any asset is directly proportional to its β_i .

Equation (8) has three testable implications. (1) The relationship between expected return on a security and its risk in any efficient portfolio m is linear. (2) β_i is a complete measure of risk of asset i in the efficient portfolio m, no other measure of the risk of i appear in relation (8) and (3) In a market of risk averse investors, high risk should be associated with higher expected return.

APPENDIX B

Let us assume that investor may take long or short position of any size in any risky asset, but there is no risk-free asset and that no borrowing and lending at riskless rate of interest is allowed. We use the analysis suggested by (Black (1972), in the world without a risk-free asset and allowed the two funds of risky assets could be identified which span the efficient portfolio set *m*.

Let every efficient portfolio consist of a weighted combination of two basic portfolios. Thus the efficient portfolios held by individual *k* is obtained by choosing proportions $xi = i = \ 1,\ 2,\ -\ N$, invested in the share of each of N available assets in order to

$$\text{Minimise} \quad \operatorname{var}(\tilde{R}_k) = \sum_{i=1}^{N}\sum_{i=1}^{N} x_{ki}x_{kj}\operatorname{cov}(\tilde{R}_i,\tilde{R}_j)\ ; \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

$$\text{s.t.} \quad E(\tilde{R}_k) = \sum_{i=1}^{N} x_{ki}E(\tilde{R}_j); \qquad \sum_{j=1}^{N} x_{kj} = 1.$$

Using Lagrange multiplies S_k and T_k , this can be solved. Taking derivative of the expression with respect to x_{ki} , we have

$$\sum_{j=1}^{N} x_{kj}\operatorname{cov}(\tilde{R}_i,\tilde{R}_j) - S_k E(\tilde{R}_i) - T_{k.} = 0, \quad I{=}1,2,\ldots..N \quad \ldots \quad \ldots \ (10)$$

This set of equations determines the value of x_{ki} . If we write D_{ij} for the inverse of covariance matrix $\operatorname{cov}(\tilde{R}_i,\tilde{R}_j)$, the solution to this set of equations may be written as,

$$x_{ki} = S_k \sum_{i=1}^{N} D_{ij} E(\tilde{R}_j) + T_k \sum_{j=1}^{N} D_{ij} \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \quad (11)$$

The subscript *k*, referring to the individual investor, appeared on the right hand side of the equation only in the multiplier S_k and T_k. Thus every investor holds a linear combination of two basic portfolios and every efficient portfolio is a linear combination of these two basic portfolios. If we normalise weight, then the above Equation (11) can be written as

$$x_{ki} = w_{kp} x_{pi} + w_{kq} x_{qi} \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \ (12)$$

In Equation (12) the symbols are defined as follows

$$w_{kp} = S_k \sum_{i=1}^{N} \sum_{j=1}^{N} D_{ij} E(\tilde{R}_j)$$

$$w_{kq} = T_k \sum_{i=1}^{N} \sum_{j=1}^{N} D_{ij} \quad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \ (13)$$

$$x_{pi} = \sum_{j=1}^{N} D_{ij} E(\tilde{R}_j) / \sum_{i=1}^{N} \sum_{j=1}^{N} D_{ij} E(\tilde{R}_j)$$

$$x_{qi} = \sum_{j=1}^{N} D_{ij} / \sum_{i=1}^{N} \sum_{j=1}^{N} D_{ij};$$

Thus we have $\sum_{i=1}^{N} x_{pi} = 1, \sum_{i=1}^{N} x_{qi} = 1, w_{kp} + w_{kq} = 1$, k=1…..L … (14)

Equation (12) shows that the efficient portfolio held by investor k consist of a weighted combination of the basic portfolio *p* and *q*. However these two portfolios are not unique. Suppose that we transform the basic portfolio *p* and *q* into two different portfolios *u* and *v*, using weights w_{up}, w_{uq}, w_{uvp}, w_{vq}. Then we have

$$x_{ui} = w_{up} x_{pi} + w_{uq} x_{qi}$$

$$x_{vi} = w_{vp} x_{pi} + w_{vq} x_{qi} \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \ (15)$$

For solving Equation (15) for x_{pi} and x_{qi} let us write the resulting coefficient wpu, wpv, wqu, wqv. Then we will have,

$$x_{pi} = w_{up} x_{ui} + w_{pv} x_{vi}$$

$$x_{qi} = w_{qu} x_{ui} + w_{vq} x_{vi} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (16)$$

Substituting Equation (16) into Equation (12), we can write the efficient portfolio *k* as a linear combination of the new basic portfolios *u* and *v* as follows

$$x_{ki} = w_{ku} x_{ui} + w_{kv} x_{vi} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (17)$$

In Equation (17) the two weights sum to one.

Thus the basic portfolios *u* and *v* can be any pair of different portfolios that can be formed as weighted combination of the original pair of basic portfolios *p* and *q*. Every efficient portfolio can be represented as a weighted combination of the portfolios *u* and *v*, but they need not be efficient themselves.

Portfolio *p* and *q* must have different β's, if it is to be possible to generate every efficient portfolios as a weighted combination of these portfolios. But if they have different β's, then it will be possible to generate new basic portfolios *u* and *v* with arbitrary β's, by choosing appropriate weights. Let us choose weights such that

$$\beta_u = 1; \beta_v = 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (18)$$

Multiplying Equation (12) by the fraction x_{mk} of the total wealth held by investor *k* and summing over all investors *k*= 1, 2,…..*L*, we obtain the weights x_{mi} of each asset in the market portfolio

$$x_{mi} = \left(\sum_{k=1}^{L} x_{mk} w_{kp} \right) x_{pi} + \left(\sum_{k=1}^{L} x_{km} w_{kq} \right) x_{qi} \quad \dots \quad \dots \quad \dots \quad (19)$$

Since market portfolio is weighted combination of portfolio *p* and *q*, since β_m is one, portfolio u must be a market portfolio Thus we can rename portfolio *u* and *v* by portfolio m and *z*, we can write the return on an efficient portfolio *k* as a weighted combination of the return on portfolio m and *z*. The coefficient of return on portfolio

$$\tilde{R}_k = \beta_k \tilde{R}_m + (1 - \beta_k) \tilde{R}_z \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (20)$$

Taking expectations

$$E(\tilde{R}_k) = E(\tilde{R}_z) + \beta_k (E(\tilde{R}_m) - E(\tilde{R}_z)) \quad \dots \quad \dots \quad \dots \quad \dots \quad (21)$$

The above equation says that expected return on an efficient portfolio *k* is a linear function of β_k. Thus we can see that corresponding relationship when there is a riskless asset and riskless borrowing and lending is allowed in Equation (8)

The same Equation (21) applies to individual assets as well as to efficient portfolios. For asset *i*, from Equation (10), we get

$$\mathrm{cov}(\tilde{R}_j, \tilde{R}_k) - \mathrm{cov}(\tilde{R}_J, \tilde{R}_k) = S_k[E(\tilde{R}_i) - E(\tilde{R}_j)] \quad \ldots \quad \ldots \quad (22)$$

Since the market is an efficient portfolio, we can put m for k, and since *i* and *j* can be taken to be portfolio as well as assets, we can put *z* for *j*, then the equation becomes

$$\mathrm{cov}(\tilde{R}_i, \tilde{R}_m) = S_m[E(\tilde{R}_i) - E(\tilde{R}_z)] \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (23)$$

Rewrite Equation (23) as

$$E(\tilde{R}_i) = E(\tilde{R}_z) + [\mathrm{var}(\tilde{R}_m)/S_m]\beta_i \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (24)$$

Putting *m* for *i* in Equation (24)

$$\mathrm{var}(\tilde{R}_m)/S_m = E(\tilde{R}_m) - E(\tilde{R}_z) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

So Equation (24) becomes

$$E(\tilde{R}_i) = E(\tilde{R}_z) + \beta_i(E(\tilde{R}_m) - E(\tilde{R}_z)) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (26)$$

Thus the expected return on every asset, even when there is no risk-free asset and no risk-free borrowing is allowed, is a linear function of β. Comparing Equation (26) with Equation (8), we can see that the introduction of risk-free asset simply replaces $E(\tilde{R}_z)$ with R_f.

The above equation holds for any asset and thus for any portfolio. Setting $\beta_i = 0$ we see that every portfolio with beta equal to zero must have the same expected return as portfolios. Since return on portfolio *z* is independent of m, and since weighted combination of portfolio *m* and *z* must be efficient, *z* must be the minimum variance zero beta portfolio.

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ABSTRACT

The main objective of this study is the review of the conceptual framework of asset pricing models and discusses their implications for security analysis. The study includes the theoretical derivation of equilibrium model, usually referred to as capital asset pricing model (CAPM). This model was developed almost simultaneously by Sharpe (1964), Treynor (1961), while Lintner (1965) and Mossin (1966) and Black 1972) have extended and clarified it further. The variation through time in expected returns is common in securities and in related in plausible ways to business conditions. Therefore modified version of the asset-pricing model, known as conditional capital asset pricing model (CCAPM) is derived from static CAPM. An alternative equilibrium asset-pricing model, called the arbitrage asset pricing theory (APT) was developed by Ross (1976). The fundamental principles underlying the arbitrage prong theory are also discussed the empirical literature is reviewed and the critical analysis of empirical and theoretical model are provided.