RISK MANAGEMENT IN FINANCIAL SERVICES SECTOR - APPLICABILITY AND PERFORMANCE OF VAR MODELS IN PAKISTAN

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Abstract

Sound risk management practices by financial institution are critical to the stability of the institutions and to the sustainability of economic growth. Motivated by McNeil and Frey (2000), we measure market risk based on the Value-at-Risk (VaR) approach for the tail of the conditional distribution of KSE index return series over the period Jan 2001 - June 2012. We estimate the conditional quantiles of the loss distribution under different distributional assumptions. Our back-testing results show that the procedure based on the Extreme Value Theory performs better than methods which ignore the heavy tails of the innovations or the heteroskadasticity in returns. We also find that the analysis of Pre- global financial crisis (GFC) and Post-GFC crisis period leads to suggest dynamic risk measures with different characteristics.

Keywords: Value at Risk, GARCH models, Extreme value theory, Back-testing, Global Financial Crisis

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I. Introduction

Financial services sector has become a major driver of economic growth in developing countries through innovation in response to the forces of globalization and technology. Sound risk management practices by financial institution are critical to the stability of the institutions and to the sustainability of economic growth. In the management of market risk, the Value-at-Risk (VaR) models have become the standard practice; VAR is defined as the maximum possible loss to the value of financial assets with a given probability over a certain time horizon. However, the task of implementing the VaR approach still remains a challenge as the empirical return distributions are found to be fat tailed and skewed in contrast to the normal distribution as assumed in the theoretical models. An extensive literature in finance (e.g., Nassim Taleb's *The Black Swan*) underscores the importance of rare events in asset pricing and portfolio choice. These rare events may materialize in the shape of a large positive or negative investment returns, a stock market crash, major defaults, or the collapse of risky asset prices.

In order to address the problems of heavy tails, VaR risk measures based on *Extreme Value Theory* (EVT) have been developed to model tail risk. EVT allows us to model the tails of distributions, and to estimate the probabilities of the extreme movements that can be expected in financial markets. The basic idea behind EVT is that in applications where one is concerned about risk of extreme loss, it is more appropriate to separately model the tails of the return distribution. At a more fundamental level, the issue is whether or not the return distributions remain stable over time. EVT's usage to model risk, however, still assumes that the probability distribution parameters extracted from the historical data are stable.

Application of the EVT to the developing countries' financial markets poses special challenges. In particular, these offer only limited data histories while EVT uses extreme observations which typical are rather rare. In addition, the return generating processes may not be stable due to the evolving institutional and regulatory environment. Pakistan offers an instructive case study since due to its turbulent political and economic environment its equity market has experienced very high volatility and incidence of extreme returns, thereby, providing a richer dataset. Yet the country has one of the oldest stock markets among the developing countries with well established institutions and regulatory structure. The backdrop of the global financial crisis of 2007-09 (GFC) also provides us with an historical experiment to examine the tails of stock return distributions. During the GFC period stock market volatility increased many folds and large swings in the stock prices were observed with an unprecedented frequency, thus, providing us with a rich data set for applying EVT.

Up till now only a few recent studies have examined the impact of GFC on the stock market behavior. Among these, Uppal and Mangla (2012) compare the tail distributions of stock returns for the pre- and post Global Financial Crises periods. There have, however, been a number of studies using EVT following previous stock markets crashes and periods of high volatility in the developed as well as the emerging markets. For example, Gencay and Selcuk (2004) employ VaR models using EVT to a sample of emerging markets after the Asian financial crisis of 1998. Onour (2010) presents estimation of extreme risk in three stock markets in the Gulf Cooperation Council (GCC) countries, Saudi Arabia, Kuwait, and United Arab Emirates, in addition to the S& P 500 stock index, using the Generalized Pareto Distribution (GPD). Djakovic, Andjelic and Borocki(2011) investigates the performance of extreme value theory with the daily stock index returns for four different emerging markets, the Serbian, Croatian, Slovenian and

Hungarian stock indices. Bhattacharyya and Ritolia (2008) suggest a Value-at-Risk (VaR) measure for the Indian stock market based on the Extreme Value Theory for determining margin requirements for traders. Iqbal, Azher and Ijza (2010) computed the VaR by considering 17 years data of Karachi Stock Exchange (KSE) using four different parametric methods and two non-parametric methods. Qayyum and Nawaz (2010) use extreme value theory to compute VaR of return series for KSE 100 during 1993-2009. Nawaz and Afzal (2011) computed the VaR using Historical Simulation, Pro and Risk Metrics method. He concluded that the Pakistan Stock Exchange VaR system is more effective than Slab system.

This study examines the performance of the extreme loss risk estimates for the equity market of Pakistan, the Karachi Stock Exchange (KSE). Various techniques of measuring market risk based on the VaR approach are evaluated for the tail of the conditional distribution of KSE index return series over the period January 2001 - June 2012. Motivated by the approach suggested by McNeil and Frey (2000), we model the conditional quantiles of the loss distribution under the dynamic framework. Our back-testing results show that the procedure based on the EVT is applicable for modeling market risk, and seems to perform better than methods which ignore the heavy tails of the innovations or the hetero-skadasticity in returns. First part of our work considered the whole sample period, whereas in second part we modeled the dynamic VaR measure for pre-crisis and post crisis periods separately.

Our study addresses, firstly, the issue whether EVT can help in measuring and managing tail risk in the emerging markets. Secondly, it addresses the issue of the stability of parameters. Even if the EVT does adequately describe the extreme return distribution, its applicability would be much restricted if the parameters of the distribution are not stable. Our study finds that the KSE is characterized by tail-distributions with significantly different characteristics in different periods, and suggests that the usefulness of Static EVT methods in assessing market risk in emerging markets may be rather limited.

II. EVT Models of Distribution Tails

Value at Risk (VaR) is a high quantile (typically the 95th or 99th percentile) of the distribution of negative returns and provides an upper bound on losses with a specified probability. However, classical VaR measures based on the assumption of normal distribution of the stock returns underestimate risk as the actual return distributions exhibit heavier tails. One alternative to deal with the non-normality of the financial asset distributions has been to employ historical simulation methodology which does not make any distributional assumptions, and the risk measures are calculated directly from the past observed returns. However, the historical approach sill assumes that the distribution of past stock prices will be stable in the future.

Another approach is to use Extreme Value Theory to construct models which account for such thick tails as are empirically observed. According to EVT, the form of the distribution of extreme returns is precisely known and independent of the process generating returns (Fisher and Tippet, 1928); Also see Longin (1996), Longin and Solnik (2001) and Chou (2005), and, Diebold et al. (2000) for a note of caution. The family of extreme value distributions can be presented under a single parameterization, known as the Generalized Extreme Value (GEV) distribution (Embrechts at el., 1997).

There are two ways of modeling extremes of a stochastic variable. One approach is to subdivide the sample into m blocks and then obtain the maximum from each block, the *block maxima method*. The distribution of block maxima can be modeled by fitting the GEV to the set of block maxima. An alternative approach takes large values of the sample which exceed a certain threshold u, the peak-over-threshold (POT) approach. The distribution function of these *exceedances* is then obtained employing fat-tailed distributions models such as the Generalized Pareto Distribution (GPD). However, the POT approach is the preferred approach in modeling financial time series.

In this paper, we use a semi-parametric approach based on the Hill estimator (Hill, 1975) for the tail index. We assume that the distribution function underlying the data satisfies, for some positive constant *C*,

$$1 - F(x) \sim \left(\frac{x}{C}\right)^{-1/\gamma} \text{ , as } \; x \to \infty \; \text{ with } \; \gamma > 0.$$

Weissman (1978) proposed the following estimator of a high quantile (i.e., the Value-at-Risk):

$$\widehat{z_q} = X_{n-k+1}\left(\frac{k}{n(1-q)}\right)^{\hat{\xi}} \qquad (1)$$

where $X_{n-k+1}$is the *k-th* top order statistic of the *n* number of observations, $\hat{\xi}$ be any consistent estimator for ξ and $\widehat{z_q}$stands for quantile function at a given confidence level q.

Among various choices, for heavy tails, the classical semi-parametric Hill tail index estimator used in equation (1) is given by the functional expression

$$\hat{\xi} = \frac{1}{k}\sum_{i=1}^{k} ln\left(\frac{X_{n-i+1}}{X_{n-k}}\right) \qquad (2)$$

The important step in this procedure is to determine the threshold (i.e., X_{n-k+1}) for identifying the tail region. It involves a trade-off: a very high threshold level may provide too few points for estimation, while a low threshold level may render a poor approximation. Several researchers, (e.g., McNeil, 1997, 1999) suggest employing a high enough percentile as the threshold. We consider 95^{th} percentile as the threshold, as is typically recommended.

III. Dynamics of Volatility and Conditional mean

Although EVT is an appropriate approach for modeling the tail behavior of stock returns, the assumption of constant volatility is contradicted by the well documented phenomenon of volatility clustering i.e., large changes in assets values are followed by large changes in either direction. Hence, a VaR calculated in a period of relative calm may seriously underestimate risk in a period of higher volatility.[1] The time varying volatility was first modeled as a ARCH (q) process (Bollerslev et al., 1992) which relates time t volatility to past squared returns up to q lags. The ARCH (q) model was expanded to include dependencies up to p lags of the past volatility. The expanded models, GARCH (p,q) have become the standard methodology to incorporate dynamic volatility in financial time series (see Poon & Granger (2003)). Similarly the auto-correlation of returns is significant in many situations and there is a need to also incorporate the ARMA(m,n) structure in the model. Let $(X_t, t \in \mathbb{Z})$ be a strictly stationary time series representing daily observations of the negative log returns on a financial asset price. The dynamics of the model has the following specification:

$$X_t = \mu_t + \sigma_t Z_t,$$

where $\mu_t = \varphi X_{t-1}$ and $\sigma^2_t = w + \alpha(X_{t-1} - \mu_{t-1})^2 + \beta\, \sigma^2_{t-1}$

with $w, \alpha, \beta > 0$, and $\alpha + \beta < 1$,

[1]See Hull & White, 1998. Acknowledging the need to incorporate time varying volatility VaR models employ various dynamic risk measures such as the Random Walk model, the GARCH, and the Exponentially Weighted Moving Average (EWMA).The Riskmetrics model uses the EWMA.

where σ_t is the volatility of the return on day t, μ_t is the expected return and (X_t) is the actual return. We assume μ_t and σ_t are measurable with respect to $\mathcal{G}_{t-1}$, the information about the return process available up to time *t-1*. The stochastic variable, Z_t represents the residuals or the innovations of the process, and is assumed to be independently and identically distributed.

Let $F_X(x)$ denote the marginal distribution of (X_t) and let $F_{(X_{t+1}\,|\,\mathcal{G}_t)}(x)$ denote the 1-step predictive distribution of the return over the next day, given knowledge of returns upto and including day *t*. We're interested in estimating quantiles in the tails of these distributions. For $0 < q < 1$, an unconditional quantile is a quantile of the marginal distribution denoted by

$$x_q = \inf\{x \in \mathbb{R}: F_X(x) \geq q\}$$

and a conditional quantile is a quantile of the predictive distribution for the return over next day denoted by

$$x_q^t = \inf\{x \in \mathbb{R}: F_{(X_{t+1}\,|\,\mathcal{G}_t)}(x) \geq q\}\,, \quad \text{where}$$

$$F_{(X_{t+1}\,|\,\mathcal{G}_t)}(x) = P\{\sigma_{t+1}Z_{t+1} + \mu_{t+1} \leq x |\, \mathcal{G}_t\} = F_z((x - \mu_{t+1})/\sigma_{t+1}\,),$$

and thus

$$x_q^t = \mu_{t+1} + \sigma_{t+1}z_q \qquad (3)$$

where z_q is the upper qth quantile of the marginal distribution of Z_t , which does not depend on *t*.

IV. Hypothesis, Data and Methodology

In this paper, we focus on the extreme returns experienced on the Karachi Stock Exchange's KSE100 index for the period January 1, 2001 to June, 2012 or 2972 observations for over 10 years. During the period the market experienced a number of political and economic shocks, including the 9/11 terrorist attack, and the Global Financial Crisis. The stock returns r_t are measured as the first negative log differences of the stock index; $r_t = -\ln(\text{Index}_t/\ \text{Index}_{t-1})$, since we are interested in the upper tail of loss distribution.

Following the approach suggested by McNeil and Frey (2000), we apply EVT to the residuals extracted from a GARCH model. Our estimation can be summarized as a two step procedure: (i) An AR(1)-GARCH(1,1) model is fitted to the historical return data by pseudo maximum likelihood method. The residuals of this model are extracted; (ii) Hill estimation procedure is employed on the right tail of the standardized residuals to compute $\text{VaR}(Z)_q = \widehat{z_q}$. Finally the estimated dynamic or conditional VaR using eq. (3) is:

$$\hat{x}_q^t = \widehat{\mu_{t+1}} + \widehat{z_q}\widehat{\sigma_{t+1}} \qquad (4)$$

We also apply various tests and report test value and the achieved p-value to verify our estimation procedure. First we apply the Augmented Ducky Fuller test to see whether the negative return series is stationary or not. After the stage (i), we apply ARCH-LM test with the null hypothesis that there is no autocorrelation among the residuals and squared residuals. Similarly after the stage (ii) of our estimation, we consider the Cramer-von Mises (W^2), Watson (U^2) and Anderson-Darling (A^2) criteria for judging the goodness of fit of the cumulative distribution function for the estimated parameters.

We further backtest the method on historical series of negative log-returns$\{x_1, x_2, \ldots, x_n\}$ from January 2001 - June 2012.We calculate $\hat{x}_q^t$ on day *t* in the set *T={m,m+1,.....,n-1}* using a time window of *m* days each time. Similar to McNeil and Frey (2000), we set *m*=1000, but we consider 50 extreme observations from the upper tail of the innovation distribution i.e., we fix *k*=50 each time. On each day$t \in T$, we fit a new AR(1)-GARCH(1,1) model and determine a new Hill tail estimate. We compare $\hat{x}_q^t$ with x_{t+1} for $q \in \{0.95, 0.975, 0.99, 0.995\}$. A violation is said to occur whenever $x_{t+1} > \hat{x}_q^t$. We then apply a one-sided binomial test based on the number of violations for the model adequacy.

We also compare the method with four other well-known parametric methods of estimation. First one is the Static normal method in which returns are assumed to be normally distributed and the VaR is calculated as the *qth* upper quantile from the normal distribution. Second one is the Dynamic or Conditional Normal in which AR(1)-GARCH(1,1) model with normal innovations is fitted by the method of maximum likelihood to the return data and $\hat{x}_q^t$ is estimated. Third one is the Conditional t in which innovations are assumed to have a leptokurtic distribution and the AR(1)-GARCH(1,1) model with t-innovations is fitted to the return data. Last one is the Static EVT method in which returns are assumed to have fat-tailed distribution and extreme value theory is applied to the upper tail of the returns.

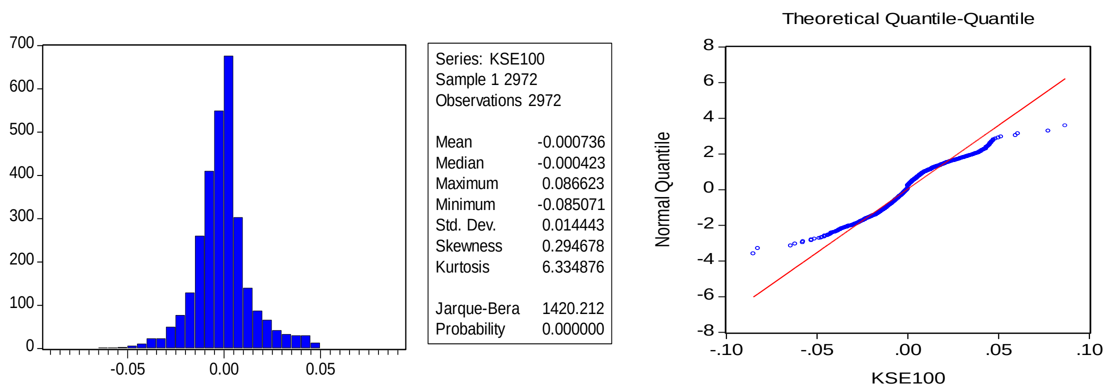
Following the idea suggested by Uppal and Mangla (2012), we also apply the methodology to model the dynamic Value-at-Risk according to time-line of the progression of the GFC. In this regard, we mark the onset of the down turn in the stock markets as the first of July, 2007. This divides our study in two sub-periods of 1158 observations each and model the dynamic risk measure for two periods separately.

V. Results

i. Modeling

We use EVIEWS 5.0 and R 2.15.1 for the analysis. The table in the following exhibit provides descriptive statistics for the KSE 100 for the period covered in the study, computing market returns as negative first log differences in the index values; Rt = - ln(Index$_t$ / Index$_{t-1}$). The exhibit also shows the distribution of the returns and a QQ-plot against normal distribution.

Exhibit 1: Descriptive Statistics

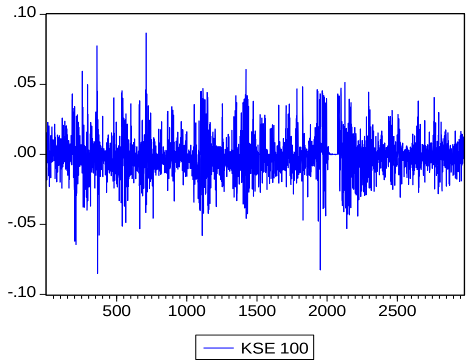


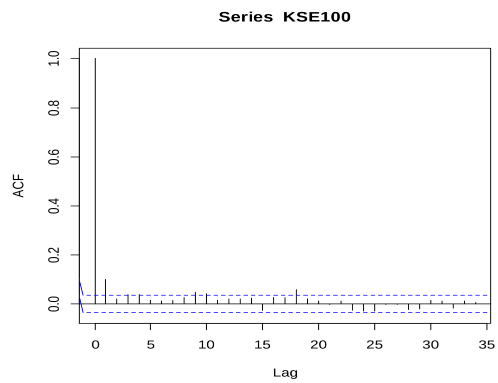
Source: Author calculation

The descriptive statistics of the stock returns clearly show that the return distributions have heavier tails than of a normal distribution. The Jarque-Bera statistic is significant even at very low levels. Hence, we reject the null hypothesis that the stock returns follow a normal distribution. High values of the

Kurtosis statistics indicate that the distributions have fat tails. The positive value of skewness indicates that the upper tail of loss distribution (i.e. the tail of interest for VaR calculation) is particularly thick. The QQ-plot also indicates the departure from normality. Therefore, nature of distributions provides support for modeling the tails of the distribution using EVT.

Figure 1: KSE 100 return series and correlogram of returns





The figure 1 clearly depicts that the large changes tend to be followed by large changes of either sign and small changes tend to be followed by small changes. This indicates that returns are not independent identically distributed and the volatility clustering phenomenon is present in the data. This suggests to employ GARCH model to estimate dynamic volitality. Similarly the ACF plot indicates that auto-correlation of returns is significant upto lag 1 and there is a need to incorporate the AR(1) component in the model to capture the effect of conditional mean. The Augmented Ducky Fuller test (given in Appendix A) strongly rejects the null hypothesis which implies that the series is stationary.

The next step is to estimate the parameters of the AR(1)-GARCH(1,1) models. The model is fitted using the pseudo-likelihood method. This means that the likelihood for a GARCH(1,1) model with normal innovations is maximized to obtain the parameter estimates $\hat{\theta} = \left(\hat{\varphi}, \hat{\omega}, \hat{\alpha}, \hat{\beta}\right)$. The results of GARCH estimation procedure are given in Table 2. All the coefficients of the volatility and mean equations are significant. The Durbin Watson Statistics are within the acceptable range implying that the model's specification is tenable.

Table 2: Results from AR-GARCH Estimation

Dependent Variable: NEG-RETURN
Method: ML - ARCH (BHHH) - Normal distribution
Included observations: 2971 after adjustments

	Coefficient	Std. Error	z-Statistic	Prob.
Mean Equation				
AR(1)	0.0641	0.0190	3.3650	0.0008
Variance Equation				
C	9.33E-06	6.89E-07	13.5508	0.0000
RESID(-1)^2	0.1486	0.0102	14.5711	0.0000
GARCH(-1)	0.8073	0.0104	77.4652	0.0000
Durbin-Watson stat	1.9263			

Source: Author calculation

We ran the AARCH-LM residual test (given in Appendix A) for the standardized residuals extracted from AR(1)-GARCH(1,1) model and found no evidence against the independent identically distributed (iid) hypothesis for the residuals. We conclude that the fitted model is tenable. The descriptive statistics and QQ-plot of standardized residuals again indicate the departure from normality and fat right tail.

Next the ordered residuals are used to estimate the right tail index using equation (2). Table 3 provides results for the estimation of parameters on the right tail of the distribution. We fix the threshold by rounding off 95th percentile value.

Table 3: Results from tail-index Estimation

Method: Maximum Likelihood (Exact Solution)				
Parameter	Value	Std. Error	z-Statistic	Prob.
Threshold, β	2.159700			
Tail Index (1/ξ)	3.470885	0.454797	7.631731	0.0000
Log likelihood	-52.88859	Mean dependent var.		2.997578
No. of Coefficients	1	S.D. dependent var.		0.959473

Source: Author calculation

We ran the Goodness of fit test (given in Appendix A) to see whether the fitted model to the right tail of the innovation distribution (which represents losses) is appropriate and we found that the p-values for all three different tests are insignificant. It implies that the parameter estimates obtained in Table 4 are tenable.

After specifying our models completely by estimating the parameters, we can now calculate the dynamic VaR estimates by using Eq. (3) . We first calculate the 95th percentile of innovations. The value of $\widehat{z_{0.95}}$ =VaR(Z)$_{0.95}$ is found out be 2.16346. Using Eq. (3), our dynamic VaR specification for the t+1 day is:

$$VaR_{0.95}^{t} = \widehat{\mu_{t+1}} + 2.16346\ \widehat{\sigma_{t+1}} \quad (5)$$

where $\widehat{\mu_{t+1}}$ and $\widehat{\sigma_{t+1}}$ are conditional GARCH estimates of mean and volatility respectively.

ii Backtesting

We then proceed to conduct back-tests using methodology explained in the section IV. Table 4 provides the back testing results with theoretically expected number of violations and the number of violations using Dynamic EVT or GARCH-EVT model, Static EVT model, a GARCH model with Student t innovations, a GARCH-model with normally distributed innovations and Static normal model in which returns are assumed to be normally distributed. It is found that Dynamic EVT correctly estimates all the conditional quantiles, since the p-value is insignificant at all levels. The method is closest to the mark in 3 out of 4 cases. Static EVT method fails at 99% and 99.5%. Dynamic-t fails at 97.5%, but performs well at higher levels i.e., it is closest to the mark at 99% and 99.5%.This indicates that GARCH model with t- innovations can also provide a good fit. Dynamic normal fails at 99.5%, whereas Static normal fails at all levels except 95%. The results shows that the Value-at-Risk models based on the time-varying volitality works better than the Static models. The distribution of the tails of innovations are modeled better using Extreme Value theory or t-distribution instead of normal distribution.

Table 4: Backtesting Results

Quantile	95%	97.5%	99%	99.5%
Length of Test	1972	1972	1972	1972
Expected # violations	99	49	20	10
DYNAMIC-EVT				
Observed # violations	96	49	18	6
p-value	(0.4199)	(1.0000)	(0.4046)	(0.1386)
STATIC-EVT				
Observed # violations	78	46	6	1
p-value	(0.0164)	(0.3503)	(0.0003)	(0.0001)
DYNAMIC-t				
Observed # violations	122	63	22	10
p-value	(0.0107)	(0.0321)	(0.3323)	(0.8730)
DYNAMIC-NORMAL				
Observed # violations	86	55	25	21
p-value	(0.1039)	(0.2236)	(0.1405)	(0.0013)
STATIC NORMAL				
Observed # violations	111	79	51	30
p-value	(0.1108)	(0.0000)	(0.0000)	(0.0000)

Source: Author calculation

iii Pre and Post Crisis VaR estimation

Following the idea suggested by Uppal and Mangla (2012), next our study spans a time period from January, 2003 to June, 2012, evenly divided in two sub-periods of 1158 observations each, as follows:

1. Pre Crisis Period: 01/22/2003 to 06/29/2007
2. Post Crisis Period: 07/02/2007 to 06/12/2012

Table 5 provides the descriptive statistics and tests for equality of means and variances of negative log returns in Pre crisis and Crisis (or Post crisis) periods. The mean, extreme values and standard deviation seems to be same in both periods. However, the test for equality of variances in Pre Crisis and Post Crisis periods indicates that the structure of dispersion is different in both periods. This justifies to study the characteristics of both periods separately. Similarly both returns series have a positive skewness, but the distribution in Pre Crisis period is skewed more towards right than Post Crisis period. However, the measure of kurtosis indicates that the tails of Post Crisis return distribution is heavier than the tails of Pre Crisis return distribution. The significant value of Jarque-Bera statistic indicates the rejection of null hypothesis that returns follow the normal distribution in both cases.

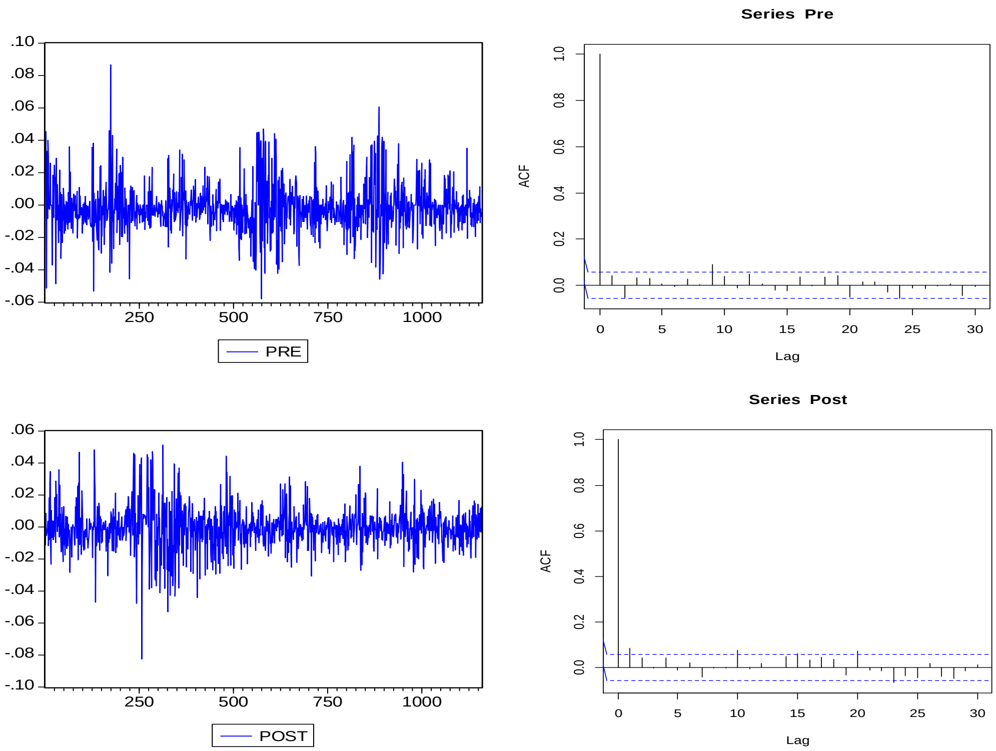
Table 5: Summary Statistics for Pre and Post Crisis returns

Series	Mean	Maximum	Minimum	Std.Dev.	Skewness	Kurtosis	Jarque-Bera
Pre Crisis	-0.001377	0.086623	-0.057967	0.015157	0.564862	5.599601	387.6502
Crisis	-0.000502	0.051349	-0.082547	0.013165	0.014111	6.346533	540.4040
T-Test for equality of means		Value	Probability	F-test for equality of Variances		Value	Probability
		1.484125	0.1379			1.325516	0.0000

Source: Author calculation

As a preliminary check, the Augmented Ducky Fuller test (given in Appendix B) indicates that both the returns series has no unit roots which implies that pre and post crisis series are stationary. Interestingly the correlograms given in Figure 2, indicates that the auto-correlation of Pre-Crisis period returns is insignificant, whereas Pre-Crisis period indicates the presence of AR(1) component. This implies it is necessary to introduce the ARMA structure for modeling the Post Crisis period. However, both the return series clearly indicates the presence of GARCH effects.

Figure 2: Returns and Correlogram of Pre and Post Crisis series



The behavior of returns and correlogram leads to propose the GARCH models given in Table 6. We again ran the ARCH-LM residual tests (given in Appendix B) for the standardized residuals extracted from fitted models and found no evidence against the independent identically distributed (iid) hypothesis for the residuals. However, the descriptive statistics and QQ-plot of standardized residuals again indicate the departure from normality and fat right tails.

Table 6: Results from AR-GARCH Estimation

	PRE-CRISIS PERIOD			CRISIS PERIOD		
	Coeff	z-Stat	Prob.	Coeff	z-Stat	Prob.
Mean Equation						
C	-0.0022	-6.7527	0.0000			
AR(1)				0.0663	2.1512	0.0315
Variance Equation						
C	1.04E-05	5.6549	0.0000	5.36E-06	5.0192	0.0000
RESID(-1)^2	0.1752	8.4535	0.0000	0.09324	8.00052	0.0000
GARCH(-1)	0.7754	35.0254	0.0000	0.87489	60.6717	0.0000
Durbin-Watson stat	1.9048			1.9618		

Source: Author calculation

The next step is to model the right tails of innovations of Pre and Post Crisis period. We consider ordered residuals and the Hill method given in equation (2) to estimate the tail index. Table 7 provides results for the estimation of parameters on the right tails of the distribution with threshold being set by rounding off 95th percentile value. It also provides various goodness of fit tests which indicates that fitted models are appropriate.

Table 7: Results from tail-index Estimation and Goodness of Fit tests

KSE 100	ESTIMATION OF EMPIRICAL DISTRIBUTION					
	PRE- CRISIS PERIOD			CRISIS PERIOD		
Test of Distribution Fit	*Value*		*Prob*	*Value*		*Prob*
Cramer-von Mises (W2)	0.125642		0.2116	0.07799		0.4628
Watson (U2)	0.105444		0.1710	0.057925		0.5048
Anderson-Darling (A2)	1.019529		0.1069	0.634751		0.3235
Parameter Estimate	*Value*	*z-Stat*	*Prob.*	*Value*	*z-Stat*	*Prob.*
Threshold, β	2.2301			2.194		
Tail Index (1/ξ)	4.2475	4.8497	0.0000	3.9022	4.5361	0.0000

Source: Author calculation

After specifying our models completely by estimating the parameters, we can now calculate the dynamic VaR estimates by using Eq. (3) for Pre-Crisis and Post-Crisis periods separately.

We first calculate the 95 percentile VaR for Pre-Crisis period .The value of $\widehat{z_{0.95}}$ =VaR(Z)$_{0.975}$ is found out be 2.242052. Using Eq. (3), our dynamic VaR specification for Pre-Crisis returns is:

$$VaR^t_{(PRE)0.95} = \hat{C} + 2.242052\ \widehat{\sigma_{t+1}}, \quad (5)$$

where $\widehat{C}$ is the estimate of constant .

We now report the 95 percentile VaR for Post-Crisis period .The value of $\widehat{z_{0.95}}$ =VaR(Z)$_{0.975}$ is found out be 2.256842. Using Eq. (3), our dynamic VaR specification for Post Crisis returns is:

$$VaR^t_{(POST)0.95} = \widehat{\mu_{t+1}} + 2.256842\ \widehat{\sigma_{t+1}}, \quad (6)$$

VI. Summary and Conclusions

A major shortcoming of various VaR measures has been that the actual return distributions exhibit fatter tails than the normal distribution would specify. As a remedy the *extreme value theory* (EVT) has been employed to explicitly incorporate extreme values, and modifying VaR accordingly. Typically, there would be limited number of extreme observations during a given period, which makes it hard to test and apply EVT as parameters are estimated with low levels of confidence. The equity market in Pakistan provides an excellent case to study the applicability of the EVT theory in a developing country. The stock market has exhibited a high degree of volatility reflecting a risky political and economic environment. The recent global financial crisis has been another source of extreme returns. The confluence of the two sources of volatility provides us with an historic experiment to test the EVT more rigorously.

We apply the EVT to the KSE100 index over a period of 11 years, 2001-2012. We find that the returns distributions are fat tailed and the General Pareto Distribution (GPD) model fits the observed distribution of extreme values well. Our back-testing exercise shows that VaR measures with dynamic adjustment for volatility clustering perform better than measures which are based on normal distribution assumption, or do not take the dynamics of volatility into account. However, we find that the estimated tail-indices of the GPD distribution vary significantly over time. The implication is that the static extreme loss estimates based on one period may not be a reliable guide to the risk of actual losses during subsequent periods, and need to be updated using a dynamic framework. We also find that the analysis of pre-crisis and post-crisis periods leads to suggest dynamic risk measures with different characteristics. The dynamic measure suggested in eq. (6) seems to be higher than the one in eq. (5). This suggests that during any financial turbulence, one needs to update the model with recent data and don't rely on the previous models. The methodology studied in this paper can be applied to any financial firm which use a holding period of one day to control internal risk at higher confidence levels.

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Appendix A

Table 1: Augmented Ducky Fuller Unit Root test

t-Statistics	-49.31500
Probability	0.0001

Exhibit 2: Descriptive Statistics of standardized residuals

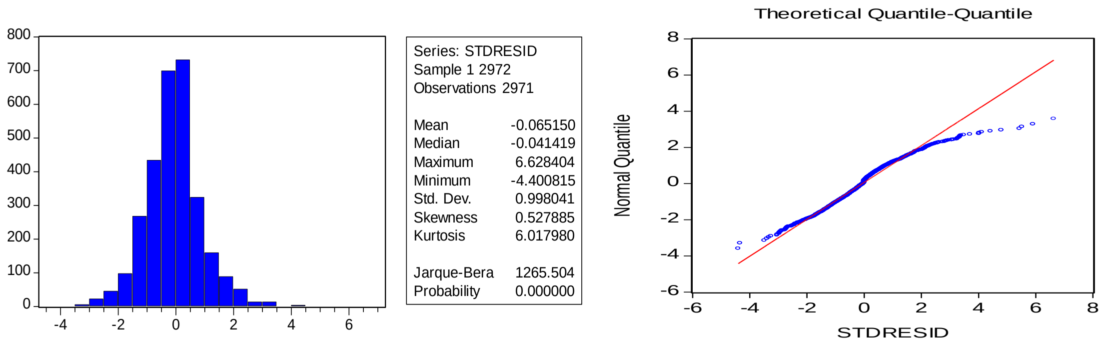

Table 3: ARCH LM residual test results

F-Statistics	0.596986
Probability	0.439792

Table 3: Goodness of Fit test results for tail index estimation

Method	Value	Probability
Cramer-von Mises (W2)	0.05709	0.63644
Watson (U2)	0.05680	0.51569
Anderson-Darling (A2)	0.43121	0.58047

Appendix B

Table 1: Augmented Ducky Fuller Unit Root test for Pre-Crisis returns

t-Statistics	-32.56522
Probability	0.0000

Exhibit 2: Descriptive Statistics for Pre-Crisis standardized residuals

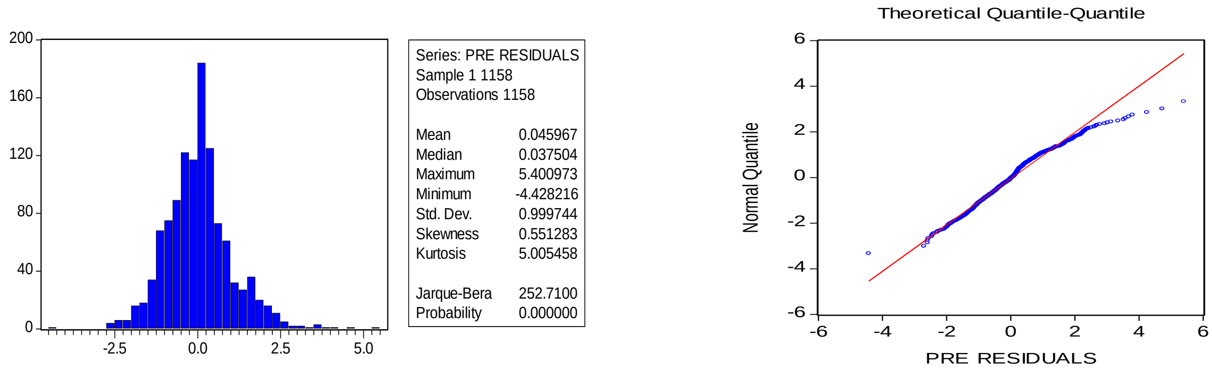

Table 3: ARCH LM residual test Pre-Crisis Returns

F-Statistic	0.324707
Probability	0.568903

Table 4: Augmented Ducky Fuller Unit Root test for Post-Crisis returns

t-Statistics	-31.9345
Probability	0.0000

Exhibit 5: Descriptive Statistics for Post-Crisis standardized residuals

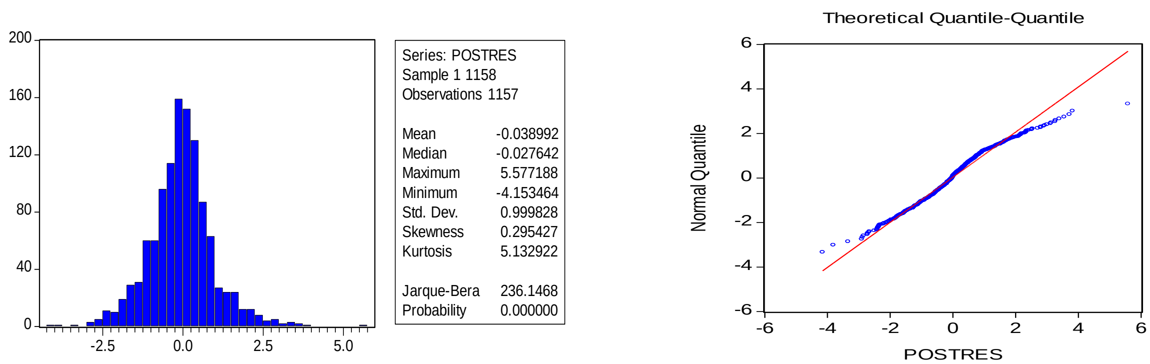

Table 6: ARCH LM residual test for Post-Crisis Returns

F-Statistic	0.000263
Probability	0.987065