Measuring Multidimensional Poverty and Inequality in Pakistan

By

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Abstract

The study focuses on poverty and inequality in Pakistan using Pakistan Social and Living Standard Measurement (PSLM) survey data of 2005-06 and 2010-11. Alkire and Foster (2007) methodology is used to develop multidimensional poverty index and multidimensional Gini Index proposed by Koen and Logu (2012) methodology for inequality. The indicators used in the analysis are equivalent per adult expenditures, maximum years of schooling, enrolment, immunization against measles, post-natal care, crowding, gas, electricity, safe drinking water and sanitation facility. Multidimensional inequality encompasses equivalent per adult expenditures, maximum years of schooling and health risk index. Poverty and inequality in uni-dimensional as well as multidimensional has declined but monetary-based measures generally do not reflect the real magnitude of poverty and inequality in Pakistan. The major contributing factors in poverty are education (28 percent) and health (28 percent). The dimensional disaggregation of poverty also showed that crowding and access to safe drinking water has greater share in poverty. The study also reviews that inequality has declined more in monetary dimension than aggregate multidimensional inequality and on the other hand inequality in health is rising in Pakistan. The high growth is found regarding inequality in the distributive health and education which also affect distribution of income inequality.

1. Introduction:

Uni-dimensional poverty is unable to capture a true picture of poverty because poverty is more than income deprivation. Multi-dimensional poverty measures have taken place of uni-dimensional poverty measures because an improvement in income does not guarantee an improvement in other dimensions like health, education and living standard. According to capability approach, poverty is not only deprivation of income rather it is deprivation of other social indicators, *i.e.* health, education and other capabilities. Poverty is not the state of deprivation of certain level of income, i.e. one dollar per day or two dollars per day but poverty is a state of multiple deprivations that poor faces. This multiplicity of dimensions leads to a broad definition of poverty. Multidimensional poverty does not fluctuate due to inflation. So, this is a relatively stable measure. Indicators reflect a relatively long term accumulation of welfare.

Alkire and Santos (2010) encapsulate certain limitations of uni-dimensional poverty. There are fundamental flaws completely concentrating upon consumption or income data while scrutinizing poverty. Consequently, Poverty is now usually estimated in multidimensional context. Although measuring MPI is a difficult task but due to the importance of MPI developed and developing countries have adopted multi-dimensional poverty estimation. First empirical implementation of capability approach has been evolved through introduction to HDI in the first Human Development Report (HDR) in 1990 by UNDP. HDI has two major flaws; firstly, it does not concentrate on most deprived population and it overlooks the dispersion of destitution among dimensions and households. Secondly, it merely focuses on averages of health, education and income at national level. To overcome erstwhile, UNDP formulated Human Poverty Index HPI in 1997 which looked like HDI, but it deals with those people who live below some threshold of income. Owing to the second curb, various

multidimensional poverty measures have been introduced (i.e. Bourgignon and Chakravarty (2003), Jayaraj and Subramnian (2010), and Alkire and Foster (2009).

People care about many features of their living along with income e.g. health, education, employment opportunities etc. These non-monetary aspects of their life are non-tradable and semi-correlated with each other. So, multidimensional nature of wellbeing is required to incorporate other dimensions comprehensively in measuring poverty and inequality. After Sen's broad capability approach, limitation of monetary analysis, and adoption of MDGs converted the attention of researchers as well as policy makers towards multidimensional measurement of wellbeing. Present study focuses on multidimensional analysis of development measures (Poverty, inequality, etc.) in Pakistan.

In Pakistan, for alleviation of poverty, different social programs, like Benazir Income Support Program (BISP) and Wasila-e-Rozgar scheme etc., have been introduced. Implementation of Poverty Reduction Strategy Papers (PRSP) approach and pledge to achieve Millennium Development Goals (MDGs) by Government of Pakistan reveal the importance of and need of poverty reduction. All poverty reduction strategies and social safety nets require the accurate analysis of poverty estimates to reach the primal objective of these programmes.

Many studies in 1990's have put a great emphasis on the relationship between economic growth, inequality and poverty like Bourguignon, Morrisson (2002) Atkinson, Brandolini (2004) and Milanovic (2005). It is considered that the combination of economic growth and inequality reduction policies are the key determinants of poverty reduction. Eradication of poverty is one of the most important objectives of all countries but the question arises, how to tackle this ambiguous problem? To answer this question idea of pro-poor growth emerged in early twenty's. Growth which favours the poor and decreases relative poverty is called pro-poor growth.

For achieving eight Millennium Development Goals, it is necessary to gauge the multidimensionality of poverty, inequality and pro-poor growth.

Main objective of this study is to measure multidimensional poverty and multidimensional inequality in Pakistan. Rest of the study is organized as section 2 gives literature review. Section 3 illustrates data and methodology. Results are discussed in section 4. Section 5 presents conclusion and policy implications.

2. Literature Review:

After Capability approach proposed by Amartya Sen, many authors and institutions started to develop methods for multidimensional poverty estimation and comparisons. The Millennium Development Goals upraised the importance of multidimensional poverty. A basic chore for research has been to develop a coherent framework for measuring multidimensional poverty that has built on uni-dimensional techniques and can be extended on other dimensions of deprivation.

Böhnke and Delhey (1999) estimated multidimensional poverty in Germany and Britain by using data sets from the survey of Breadline Britain 1990 and German Welfare 1998. Results have indicated that Britain has more unequal distribution. Britain has also low living standard.

Ahmed (2000) estimated various inequality indices such as standard deviation of log income, Gini Coefficient, Atkinson Index, coefficient of Variance and Theil Index at individual and household level for Pakistan. The research applied Pakistan Integrated Household Survey (PIHS) 1992-93 data for analysis. The research estimated all inequality measures at rural, urban and provincial level. The finding of the study depicted that inequality among individuals was lower than inequality among the household. At individual level, Gini Coefficient was lower in urban areas (0.375) as compared to rural areas (0.384) at individual level. At provincial level, inequality was more sever in Sindh as compared to Balochistan at

household level. Rural Punjab and rural Baluchistan confronted more inequality. Moreover, urban Sindh and urban NWFP also faced high inequality.

McCulloch (2000) estimated trends in inequality in Zambia during 1991 to 1998 by using different inequality estimates like Gini Coefficient, Theil Index, Entropy measures, Standard Deviation of Log of Income and Coefficient of Variance. According to the results in Zambia, there was a declining trend in all inequality measures from 1991 to 1996 and also decline in inequality during 1996 to 1998 time period. The study also found lower inequality in urban than rural areas.

Metha (2003) measured multidimensional poverty in India for 15 states at district level and data has drawn from India Rural Development Report (NIRD), 1999 and Planning Commission Report 2000. The study has used five dimensions of poverty, i.e. the income, education, health, infrastructure and agricultural productivity. Results showed that seven districts are most deprived in which six belongs to four high income states. The major indicators of multidimensional poverty are infant mortality and literacy rate.

Anwar (2003) investigated the inequality trend for 1998 and 2001-02. The study utilized Pakistan Integrated Household Survey (PHIS) data for analysis and found, inequality has increased in Pakistan during given time period. Lorenz Curve shifted downward in the given time period and supported the rising of inequality in Pakistan. Regional analysis gave quite different picture of inequality. In urban areas inequality has declined, whereas in rural areas it has rose up.

Saboor (2004) examined inequality trend in Pakistan from 1990-91 to 2001-02. The study estimated various inequality indices like Gini coefficient, Coefficient of Variance, Atkinson Index and Theil Index. The results illustrated inequality has decline from 1990-91 to 1993-94 while inequality increased for the rest of the period.

Logu (2004) analyzed and compared multidimensional inequality in Argentine during 1991 and 2001. Average income per capita, education and health were considered as welfare indicators. The average income of the city adjusted for inflation using Consumer Price Index (CPI). Life expectancy and ratio of number of complete secondary education to the adult population were used proxies for health and education. The results of the study found higher inequality in income which became worse in 2001, while inequality in health and education decreased. Multidimensional inequality showed a declining trend during 1991 and 2001.

Justino (2005) examined multidimensional poverty in Brazil and Vietnam in 1992-93. The study utilized micro data, the Pesquisa Nacional de Amostra de Domicilios (PNAD),and Vietnam Living Standards Measurement Surveys (VLSS) of Brazil and Vietnam respectively. Three welfare indicators namely; material wellbeing (per capita income for Brazil and per capita expenditures for Vietnam), education (years spent in education by household head) and health (number of days lost attributable to illness by household head) but data on health variable was available only in VLSS. The study estimated Inequality Indices (Theil index and Generalized entropy index) and Stochastic Dominance Analysis for measuring monetary as well as non-monetary indicators. Results of the research found higher level of in equality in Brazil than Vietnam in all welfare indicators. In Brazil, income inequality is higher than income. However, multidimensional inequality was higher than income inequality in Brazil. In Vietnam health inequality and expenditure inequality rose by 30 percent and 12 percent respectively while education inequality decreased by 12 percent during 1992-93 to 1997-98. However multidimensional inequality is higher in 1992-93 as compared to 1997-98.

Esanov (2006) estimated inequality by constructing Lorenz Curve in Kazakhstan from 2001 and 2004. Findings of the study depicted a declining inequality trend for the given time period.

Alkire and foster (2007) have proposed a methodology for multidimensional poverty. This methodology is enough flexible for selection of dimensions as well as weights. This methodology is easy to understand and amenable for comparisons. Many researchers have estimated multidimensional poverty by using different methods.

Batana (2008) examined multidimensional poverty in Sub-Saharan countries by using Alkire and Foster methodology and test robustness of different cut-off "k" and changes in poverty cut-off. The study selected four dimensions: health, education, empowerment and assets, and equal weights. Results found that education contributes more than any other dimension and MPI is higher in rural areas.

Brandolini (2008) examined multidimensional poverty and inequality and compare the estimates in four countries Italy France, Germany and the United Kingdom. For analysis, European Community Household Panel (ECHP) data was used for years 1990 and 2001. Purchasing Power Parity Index (PPP) used to update unit of account (income). Self-reported health used as health proxy and income as material wellbeing for measuring multidimensional poverty and inequality. Results drawn from the study illustrate that Germany has lowest level of poverty and Inequality in income context whereas most unequal in multidimensional context while France has lowest level of multidimensional inequality. Thus multi-dimensional inequality gave true picture than uni-dimensional inequality.

Battiston et al. (2009) analysed multidimensional poverty from 1992 to 2006 in Latin American countries and employed Alkire and Foster methodology. The study has utilized six dimensions: water and shelter, child attendance at school, education of the household head, income and sanitation, and equal weights.

Alkire et al. (2009) investigated multidimensional poverty and proposed a new method for estimation by using National Health Interview Survey data for America, and divide the population in four groups Hispanic, White, African-American and Others. The dimensions

are used: income, self-reported health, health insurance and year of schooling and their cut offs are income less than poverty line, fair or poor health, lack of health insurance and lack of a high school diploma. The results represented that income poverty of African-American contributed (29 percent), poor health of Whites 26 percent, lack of health insurance of Hispanic 30 percent and lack of year of schooling of Hispanic 35 percent to multi-dimensional poverty. The study also estimated MPI for Indonesia by using dimensions: expenditure, BMI, year of schooling, drinking water, sanitation and their cut offs are expenditure less than 150,000 rupiah, BMI less than 18.5kg/m^2 , years of schooling less than 5 year, no access to clean drinking water, no access to a latrine. The results explored that 83 percent deprived in 2 dimensions and 17 percent deprived in all three dimensions.

Decancq et al. (2009) examined multidimensional poverty in several countries from 1975 to 2000. For analysis, data has drawn the World Development Indicators (2004) with five years interval. GDP per capita, life expectancy at birth, adult literacy rate and gross secondary enrolment rate were used as welfare indicator. For comparison, all indicators normalized to their mean. Two stage procedures employed for construction of Gini index. Multidimensional inequality increased over time and because population weights were not applied so countries with smaller population performed poorly than populated countries e.g. China and India. Jamal (2009) estimated the headcount ratio, poverty gap and poverty severity gap by using HIES data. Financial poverty, Physical Household Assets poverty, Human Poverty and Poor housing were the selected dimensions of the study and results illustrates that in 2004-05 54 percent of the population were multidimensional poor. In urban areas the extent of multidimensional poverty is less than in rural areas. In rural areas 69 percent were poor than in urban areas 21 percent people.

Anwar (2009) estimated inequality, by using Gini Coefficient and expenditures share by decile ratio, utilizing Household Income and Expenditures (HIES) date from 2001-02 to

2004-05. The results of the study found an increasing trend in Gini Coefficient in Pakistan. Results also depicted a worse picture of inequality in urban areas than rural areas. Expenditures share by decile ratio depicted that during the given time period, the gap between rich and poor was widening.

Naveed and Islam (2010) have analysed multidimensional poverty for Benazir Income Support Program (BISP) and developed Benazir Poverty Scored (BPS). The study has applied Alkire and foster Methodology for measuring Multidimensional Poverty at national, provincial (Punjab and KhayberPakhtonkhwa (KPK)) and district level. Data were drawn from Research Consortium of Educational Outcomes and Poverty (RECOUP) Household Survey 2005-06. This study has used twelve indicators as follows: Education, Health, Consumption, Child status, Livelihood, Housing, Electricity, Assets, Landholding, Access to safe drinking water, Sanitation and Fuel for cooking with different weights. Results showed that the probability of being poor of a rural household is 0.28 times greater than urban and probability of being poor for a Punjab resident is 1.94 times greater than resident of KPK.

Awan et al. (2011) have measured multidimensional poverty in Punjab at district level by using survey data of Multiple Indicator Cluster Survey (MICS) for the years of 2003-04. This study has selected eight dimensions, i.e. Expenditures, education, water, land, assets, sanitation, electricity and housing and used Alkire and Foster (2007) methodology for estimations. The results showed that at dimensional cut-off two most deprived cities are Muzaffargarh, Okara, Rahimyar Khan, Rajanpur, Kasur and Lodhran, whereas least deprived dictricts are Jhelum, Lahore, Rawalpindi, Sialkot and Gujranwala.

Aristei (2011) investigated multidimensional inequality Italian Survey on Income and Living Conditions (IT-SILC) for 2005 and 2008.the study used health, education, income data for measuring Tsui index (1995). Inequality widened with high value of inequality aversion parameter and low value of substitutability across individuals as well as across regions.

Awan et al. (2012) have employed Alkire and Foster methodology for measuring multidimensional poverty in Pakistan by using the data of Pakistan Social and Living Standard Measurement Survey 2005-06. Expenditures, education, water, land, assets, sanitation, electricity, empowerment and housing were the selected dimensions for analysis. Results found that in Pakistan overall 22.8 percent people were experiencing multidimensional deprivations. A regional comparison has showed that urban deprivations are11.3 percent whereas rural deprivations are 26.8 percent.

Decancq and Lugo (2012) proposed a methodology for multidimensional inequality and applied this methodology on Russian by using Russian Longitudinal Monitory Survey (RLMS-HSE) for 1995 and 2005. Education, health, expenditures and housing quality were the nominated dimensions. Expenditure per adult equivalence scale is indicator of expenditure and square root of household size was used as equivalence scale. For health dimension, composite health index has constructed by using eight indicators like heart attack, indicators of diabetes, anaemia or other health problems and life style indicators like smoking, hospitalization or medical check-up during last three months. Indicator of education has constructed by including highest level of education and number of years in post-secondary education. Housing quality index has computed by utilizing information about presence of hot water, access to a telephone connection and sewerage system. Results found that expenditure inequality has increased 10 percent to 15 percent since 1955 t0 1998 numerical value of Gini coefficient (d=2) increase from 0.39 to 0.45 whereas educational inequality has declined during the considered time period and health inequality has increased to some extent. Housing inequality has no significant change.

Rohde and Guest (2013) examined racial inequalities in US in multidimensional context. The study used the Panel Study of Income Dynamics (PSID) data from 1990 to 2007. For multidimensional inequality three welfare dimensions were used i.e. health, education and

income. Self-reported health and post government income equalized per adult were used for health, education and income respectively. The study applied Maasoumi (1986) multidimensional index and assign weights as proposed by Decancq and Lugo (2012).the results of the study showed an increasing trend in uni-dimensional as well as multidimensional inequality.

Jordá et al. (2013) estimated global inequality in multidimensional context from 1985 to 2011, where from 1985 to 2005 data have five years interval while from 2005 data have annual incidence. The data were drawn from International Human Development Indicators (UNDP, 2012) for three indicators as used in Human Development Index (HDI). The study has employed Generalized Entropy Measures for measuring multidimensional inequality based on Human Development Index (HDI). A two-step procedure has been adopted i.e. at first stage welfare indicators aggregated for individuals named as welfare function of individuals and then in second stage Generalized Entropy measure was used for measuring inequality in welfare function. Results showed different trends with different values of parameters. But in general inequality has declining trend.

Upshot, various studies employ different measures to estimate inequality, different units of analysis and different welfare indicators for analysis. Hence, the results of different studies aren't comparable. Thus for consistence inequality measurement, it is obligatory to apply the same methodology.

3. Data and Methodology:

This study has utilised Pakistan Social and Living Standard Measurement (PSLM) surveys (2005-06 & 2010-11) for analysis of multidimensional, poverty and inequality.

Two stage stratified sampling design has been adopted for this survey. At the first stage, randomly villages and enumeration blocks have been selected with probability proportional to the size (PPS) method as primary sampling units (PSUs). At the second stage, households

within PPUs have been selected as secondary sampling units (SSUs). Sample size in the survey is national representative. PSLM 2005-06 covered 15453 household and PSLM 2010-11 covered 16341 household.

This study has selected Alkire and Foster (2007) methodology for measuring multidimensional poverty and Gini index for inequality. It is easy to compute and interpret, intuitive and fulfils many desirable axioms. This study has employed, most commonly used measure, Multidimensional Poverty Index proposed by Alkire and Foster (2007). This is also called adjusted Headcount Ratio or M_0. M_0 is applicable when any one of dimension is ordinal, numerical values have no meaning, for example, qualitative in nature. Alkire and Foster Methodology (AFM) does not assume that data is continuous. Brief and simple explanation of MPI in mathematical as well non-mathematical form is given below:

Suppose N numbers of households with Đ dimensions, which are selected for MPI, are summarised by the set of achievement matrices "Q" and $Q \in Q$. Let achievement matrix $Q = [Q_{N\text{đ}}]$, nxđ, represents the achievement of Nth household in $Đ^{th}$ dimension, $\forall$ đ $= (1,2,3,4,....,Đ)$ and $\forall$ N $= (1,2,3,4,,N)$. Entry in achievement matrix $Q_{n\text{đ}}>0$ represents N^{th} individual's achievement in D^{th} dimension. Row and column vectors are represented by $Q_{N.} = (Q_{N1},\ Q_{N2},\ Q_{N3},\ Q_{N4}\dots\dots,Q_{N\text{đ}})$ and $Q_{.\text{đ}} = (Q_{1\text{đ}},\ Q_{2\text{đ}},\ Q_{3\text{đ}},Q_{4\text{đ}},\dots\dots,Q_{n\text{đ}})$ respectively. The row vector, $Q_{N.}$, denotes N^{th} household's achievements in different dimensions and column vector, $Q_{.\text{đ}}$, represents the distribution of achievements of a specific dimension among N households. $z_{\text{đ}}$ is dimensional cut-offs (poverty line) vector. "G^0", and Ҫis deprivation matrix and count vector respectively.

Suppose dimensional cut-offs (poverty line) vector is denoted by z. For each dimension Đ$z_{\text{đ}} > o$ and $z_{\text{đ}}$ denotes poverty line or dimensional cut-off for $Đ^{th}$ dimension of MPI. Deprivation matrix "G^0" is defined where $g_{n\text{đ}}$ represent $n\text{đ}^{th}$ element of deprivation matrix.

Value of each element may 0 or 1. If the person is deprived then value will be 1 otherwise 0. Mathematically it can be shown as

$$g_{n\text{đ}}^0 = \begin{cases} 1 \; if \; Q_{n\text{đ}} \; < z_{\text{đ}} \\ \\ 0 \; if \; Q_{n\text{đ}} \; > z_{\text{đ}} \end{cases}$$

Intuitively if achievement of $n\text{đ}^{th}$ household in dimension Đ is less than dimensional cut-off (poverty line) $z_{\text{đ}}$ than that household is declared as poor and in the matrix "G^0" $g_{n\text{đ}}$ entry will be 1 and if achievement of $n\text{đ}^{th}$ household in dimension Đ is greater than dimensional cut-off (poverty line) $z_{\text{đ}}$ than that household is said to be non-poor and in matrix "G^0" entry $g_{n\text{đ}}$ will be 0. By using "G^0" matrix, a column vector "C" is constructed, which represents the deprivation scores of household (the breadth of deprivation experienced by each household). Cis calculated by taking the average sum of deprivations, so the deprivation score lies between 0 and 1. The deprivation score for a household is 1 when the household experienced maximum deprivation i.e. equal to dimensions and is 0 when a household is not deprived in any dimension. In simple words, $C_n = |g_n^0|$ denotes the number of deprived dimensions experienced by n^{th} household. If dimensions are not ordinal, then we can define normalized gap matrix g^1, where each element represents relative inequality in given dimension. More precisely, $n\text{đ}^{th}$ entry of matrix g^1 is:

$$g_{n\text{đ}}^1 = \begin{cases} (z_{\text{đ}} - Q_{n\text{đ}})/z_{\text{đ}} \; if \; < z_{\text{đ}} \\ \\ 0 \qquad\qquad otherwise \end{cases}$$

Each element can adopt nonnegative value less than or equal to one. Normalized gap provides the extent of poverty in each dimension. The normalized gap generally can be denoted as g^α where the value of α must be greater than zero ($\alpha > 0$). These notations may be helpful in identifying poor and generalization of FGT measures.

Alkire and Foster proposed a dual cut-off method which lies between two extremes. In a dual cut-off approach a household is said to be poor if the household is deprived in at least k

number of dimensions; where $k = (1, 2, 3, Đ)$, if the number of deprived dimension smaller than k then the household is non-poor according to identification function. There are some characteristics of dual cut-off, i.e. firstly; an increase in the non-poor household's achievements has no impact on identification value. Secondly, increase any non-deprived dimension level, $Q_{nđ} > z_đ$, its value is unchanged; in other words, poverty status of the household remains same by an increase in any non-poor achievement. Finally, this approach can be used when data in ordinal. Let us define identification function ρ_k such that

$$\rho_k = (Q_{n,}, z) = 1 \, \text{if} \, C_n \geq k$$

and

$$\rho_k = (Q_{n,}, z) = 1 \, \text{if} \, C_n < k$$

where k is identification cut-off as described above. $k = 1$ represents identification criteria is union approach while if $k = Đ$ the identification approach is intersection approach. By employing identification function, we get set of poor household, defined as $\mathbb{z}_k = \{n : \rho_k(Q_{n\bullet}, z) = 1\}$.

Let's define a censor matrix, having zero and one values for non-poor and poor households respectively. As in uni-dimensional poverty measurement non-poor households are given zero value and else one, same is done in multidimensional poverty. We can say any non-poor person is given zero in uni-dimensional approach the same is needed to be done in multi-dimensional approach censor or replace with achievement of poverty line is zero any deprivation experienced by non-poor so that mean this deprivation has no meaning. A censor matrix $g^\alpha(k)$ is constructed from "G^α" by interchanging its nth row $g_{nđ}^\alpha$ with zeros when the household is multidimensional non-deprived i.e. $\rho_k = (Q_{n,}, z) = 0$, so that $g^\alpha(k) = \rho_k = (Q_{n,}, z)$. This censored matrix will be used in subsequent analysis e.g. the headcount ratio, the poverty gap, etc.

By using dual cut-off identification approach following measures are defined by Alkire and Foster. Firstly the headcount ratio, a simple measure which shows the percentage of multidimensional poor. Just like uni-dimensional headcount ratio, multidimensional headcount ratio $\mathbb{H} = \mathbb{h}(Q; z)$ is defined as

$$\mathbb{H} = \mathbb{q}/N$$

where

N= Total population

$\mathbb{q} = q(Q; z) = \sum_{n=1}^{N} \rho_k(Q_{n}, z)$ in set of $\mathbb{z}_k = \{n : \rho_k(Q_{n\bullet}, z) = 1\}$, i.e. number of households which are identified poor by the dual cut-off approach. Multidimensional headcount ratio violets dimensional monotonicity which is if a poor become deprived in a new dimension the headcount ratio should increase. It can't decompose dimensional contribution in poverty.

To overcome these issues Alkire and Foster proposed adjusted FGT class of measures; which includes additional information regarding the extent of deprivation faced by poor household.

The share of dimension to n[th] household's poverty is represented by $g_n(k)/đ$, hence average

deprivation share across the household A is given by $g_n(k)/Nđ$, where NĐ is the maximum

number of deprivation which a household can experience. In other words, it gives the fraction of dimensions in which average household experienced multidimensional poverty.

Adjusted class of measures has given below:

$$\mathcal{M}^\alpha = A \times \mathbb{H} = \mu(g^\alpha(k)) \text{ for } \alpha \geq 0$$

For $\alpha = 0$, FGT class of measure belong to the headcount ratio, which is given below:

$$\mathcal{M}^0 = A \times \mathbb{H} = \mu(g^0(k)) \qquad \text{For } \alpha = 0$$

This is a simple product of the average share of deprivation with raw headcount ratio. $\mathcal{M}^0$ encapsulates info on both prevalence and extant of poverty. This measure rationalizes

monotonicity; as a household experience deprivation in new dimension, A will upswing and so does $\mathcal{M}^0$. $\mathcal{M}^0$ is also easy to calculate as $\mathbb{H}$ and can be calculated with ordinal as well as cardinal data.

When $\alpha = 1$, the adjusted poverty gap can be calculated as:

$$\mathcal{M}^1 = (\mathbb{H} \times A)G = \mathcal{M}^0 \times G = \mu(g^1(k)) \qquad \text{For } \alpha = 1$$

Where the average gap is defined as:

$$G = \frac{g^1(k)}{g^0(k)}$$

$\mathcal{M}^1$ Summarizes information on the incidence of poverty, average rang of deprivation and the average gap of deprivation across households. In other words, we can say that it combines information about the adjusted headcount ratio and the average gap of poverty. It satisfies not only dimensional monotonicity but also sensitive to the depth of deprivation in all dimensions.

For $\alpha = 0$, FGT class of measure yields adjusted square poverty gap, which is given below:

$$\mathcal{M}^2 = (\mathbb{H} \times A)S = \mathcal{M}^0 \times S = \mu(g^2(k)) \qquad \text{For } \alpha = 2$$

Where average square poverty gap is defined as:

$$S = \frac{g^2(k)}{g^0(k)}$$

$\mathcal{M}^2$ Summarizes information on the incidence of poverty, the average rang and average square gap of deprivation across households. An alternate definition of $\mathcal{M}^2$ is that it is the ratio of the sum of the square of deprivation gape to maximum possible sum of square of deprivation gape or NĐ. Square poverty gape highlights the larger gaps and limits the smaller one. "Suppresses the smaller gaps and emphasizes the larger ones" (Alkire & Foster, 2011). It gives more weightage to the poorest persons. Each dimension can be assigned

different weights. For assigning weights to each dimension, a vector, named "w", is defined, keeping in view that the sum of vector w should be equal to total dimensions i.e. $\sum_{D=1}^{D} w_{đ} = Đ$.

A weighted deprivation matrix $G^{\alpha}(w_{đ}) = [g_{Nđ}^{\alpha}]$ has fabricated in such a way:

$$g_{nđ}^{\alpha} = \begin{cases} w_{đ}\,[(z_{đ} - Q_{nđ})/z_{đ}]^{\alpha} & if \quad Q_{nđ} < z_{đ} \\ \\ 0 & otherwise \end{cases}$$

Each element of $G^{\alpha}(w_{đ})$ is defined equal to weight, $G_{Nđ}^{\alpha} = g_{đ}$, if achievement below the particular dimensional poverty line, $Q_{Nđ} < z_{đ}$, i.e. household is declared as deprived in particular dimension Đ and equal to zero, $G_{Nđ}^{\alpha} = 0$, if achievement below the particular dimensional poverty line, $Q_{Nđ} \geq z_{đ}$, i.e. household is declared as non-deprived in the respective dimension. This matrix points out the poor in each dimension.

The next step of MPI is to identify the households which are multidimensionality poor for which first of all derivational score vector "s" is defined by using the weighted deprivation matrix G^{0} whose Nth element "S_{N}"$= \sum_{đ=1}^{Đ} G_{Nđ}^{0}$ denotes the sum of weighted deprivations suffered by the household "N". Formally, this can be represented as

$$S_{n} = w_{1}Đ_{1} + w_{2}Đ_{2} + w_{3}Đ_{3} + \cdots \ldots + w_{đ}Đ_{nđ}$$

The value of vector "S" lies between 1 and Đ. Then second cut-off is applied across column vector "s". Now the poverty cut-off should be a real number "κ", satisfying $0 < \kappa \leq Đ$, instead of being a positive integer. There are three approaches of identification i.e. intersection approach, union approach and dual cut-off approach. . $k = 1$ represents identification criteria is union approach, while $k = Đ$ indicate intersection approach, and if $0 < \kappa < Đ$ the identification approach will be dual cut-off. Weighted multidimensional poverty indices are similar as before.

A censor matrix $g^{\alpha}(k, \omega)$ is constructed from "$G^{\alpha}(\omega)$" by interchanging its nth row $g_{nđ}^{\alpha}(\omega)$ with zeros when the household is multidimensional non-deprived i.e. $\rho_{k} =$

$(Q_{n,}, z; \omega) = 0$, so that $g^{\alpha}(k, \omega) = \rho_k = (Q_{n,}, z; \omega)$. The adjusted FGT multidimensional poverty measure is defined as:

$$\mathcal{M}^{\alpha} = A \times \mathbb{H} = \mu(g^{\alpha}(k)) \text{ for } \alpha \geq 0$$

Value of α determines whether the poverty measure is headcount ratio, poverty gap or square poverty gap.

The most important property of Alkire and Foster methodology is that it can be decomposed by groups as well as by dimension. According to this property, the overall poverty level of population is equal to the weighted average level of poverty in subgroup. Let suppose a country has two subgroups with distribution of achievements Q_1 and Q_2 and population of subgroups is N_1 and N2 respectively. The total population is N (which is equal to N_1+N_2) so the overall poverty is defined as:

$$\mathcal{M}(Q_1, Q_2; z) = \frac{N_1}{N} \mathcal{M}(Q_1; z) + \frac{N_{12}}{N} \mathcal{M}(Q_2; z)$$

It can be extended by any number of subgroup. This property is very useful for poverty profile and poverty targeting.

Family of multidimensional FGT measures can also be decomposed by dimensions as:

$$\mathcal{M}^{\alpha}(Q; z) = \sum_{đ=1}^{Đ} \mu(g_{.đ}^{\alpha}(k)) \Big/ Đ$$

Where $g_{.đ}^{\alpha}$ is the $đ^{th}$ column of censored deprivation matrix $(g_{nđ}^{\alpha}(k)$. Contribution of dimension to overall poverty can be defined as;

$$Dimensional\ contribution = \mu(g_{.đ}^{\alpha}(k)) \Big/ \mathcal{M}(Q; z)$$

It is interpreted as the percent contribution of $đ^{th}$ dimension to overall MPI. MPI can be measured at individual or household level through AF methodology, but if we select individual as a unit of analysis there might be a problem of data availability. In Micro survey

expenditures, detail is present only on household level so it is most appropriate to analyse poverty on household level. In this study unit of analysis is household level.

3.1 Multidimensional inequality:

Fisher (1956) is one of the pioneers of multidimensional inequality, who have given the idea of multidimensional matrix. Recently Kolm (1977), Atkinson and Bourguignon (1982), Walzer (1983), Gajdos and Weymark (2005) and Decancq and Lugo (2011) also proposed multidimensional inequality indices. Kolm has proposed uniform majorization, also known as Pigou-Dalton criterion. He also has proposed directional/price majorization, in which distribution matrix is multiplied by price first and then compared with uni-dimensional distribution. Atkinson and Bourguignon proposed that multidimensional inequality which depends upon the correlation between different dimensions. Walzer introduced a theory of complex equality, according to this conception multidimensional inequality not only depend on unequal distribution among dimensions but also depend on distribution of outcomes of a specific dimension in all individuals. Hence, one can say that dimension by dimension analysis of inequality is not sufficient because it can't take into account correlation across dimensions.

Most recently Gajdos and Weymark (2005) and Decancq and Lugo developed multidimensional Gini coefficient. The difference between two methodologies is only sequence of aggregation. Gajdos and Weymark (2005) developed multidimensional inequality by double aggregation method; first aggregated the achievements across individuals and then across dimensions, this is also known as a generalized Gini coefficient. But the Decancq and Lugo (2012) aggregate in reverse direction, first aggregate across dimensions and then across individuals, this technique takes into account correlation between dimensions. So, this study has employed Decancq and Lugo multidimensional methodology for measuring multidimensional Gini coefficient.

Following notations will be helpful to understand the methodology. Suppose N numbers of household with Đ dimensions, are selected for multidimensional inequality. These are summarised by set of distributional matrices "Q" and $Q \in Q$. Let distributional matrix $Q = [Q_{Nđ}]$, nxđ, represents outcome of Nth household in $Đ^{th}$ dimension, $\forall đ = (1,2,3,4,....,Đ)$ and $\forall N = (1,2,3,4,,N)$. Entry in achievement matrix $Q_{nđ} > 0$ represents N^{th} individuals outcome in D^{th} dimensions. Row and column vectors are represented by $Q_{N.} = (Q_{N1}, Q_{N2}, Q_{N3}, Q_{N4} ...., Q_{Nđ})$ and $Q_{.đ} = (Q_{1đ}, Q_{2đ}, Q_{3đ}, Q_{4đ}, ..., Q_{nđ})$ respectively. Row vector, $Q_{N.}$, denotes N^{th} household's outcome in different dimensions and column vector, $Q_{.đ}$, represents the distribution of an outcome of a specific dimension among N households.

For measuring multidimensional inequality Gini coefficient distributional matrix can be compared with social welfare function. Social welfare function can be constructed by double aggregation. First aggregated across dimensions, we get individual welfare function, then aggregated across individual, to get social welfare function. Mathematically aggregation function can be described as:

$$\mathcal{W}_{n \times đ}: \mathcal{R}_{n \times đ}^{++} \to \mathcal{R}^{++}: Q \to \mathcal{W}_{n \times đ}(Q) = \mathcal{W}_n[\mathcal{W}_đ(Q_1), ..., \mathcal{W}_đ(Q_n)]$$

Same or different weights can be assigned to each dimension. Weights reflect the relative importance of a dimension. Weights must be assigned in such a way that the sum of all weights must be equal to one, mathematically $\sum_{d=1}^{Đ} \mathcal{W}_đ = 1$. Marginal rate of substitution can be determined by assigned weights. Equal weights show that dimensions are perfectly substitutable. Mathematically social wellbeing function can be written as:

$$\mathcal{W}_{n \times đ} = \sum_{n=1}^{N} \left[\left(\frac{r^n}{n} \right)^{\delta} - \left(\frac{r^n}{n} \right)^{\delta} \right] \left[\sum_{đ=1}^{Đ} \mathcal{W}_đ (Q_đ^n)^{\beta} \right]^{1/\beta}$$

Where $\delta > 0,$ $\mathcal{W}_đ > 0, \sum_{d=1}^{Đ} \mathcal{W}_đ = 1$

$\mathcal{W}_đ$ = weights assigned to each dimension

$$r^n = \text{rank of individual n on the basis of S}$$

$$\delta = \text{inequality inversion parameter}$$

$$\beta = \text{substitution parameter}$$

$$S = \left[\sum_{đ=1}^{Đ} \mathcal{W}_đ (Q_đ^n)^\beta \right]^{1/\beta}$$

So β mirrors the degree of substitutability among the dimensions of wellbeing. Specifically, β is related to the elasticity of the substitution σ between the dimensions, and equals 1-1/ σ. β=1, represents perfect substitutability between the dimensions of wellbeing and β = -∞, denotes dimensions are perfect complementarity; at the extreme, individuals are judged by their worst outcomes.

Now the multidimensional Gini index can be defined as the fraction of an aggregated amount of dimensions of a distributional matrix by aggregated amount of dimensions if dimensions are equally distributed. It is analogous to uni-dimensional Gini index which is also ratio of aggregated gap of income to the equally distributed income. So multidimensional Gini is defined as below:

$$I(Q) = 1 - \frac{\sum_{n=1}^{N} \left[\left(\frac{r^n}{n} \right)^\delta - \left(\frac{r^n}{n} \right)^\delta \right] \left[\sum_{đ=1}^{Đ} \mathcal{W}_đ (Q_đ^n)^\beta \right]^{1/\beta}}{\left[\sum_{đ=1}^{Đ} \mathcal{W}_đ \left(Q_đ^n \right)^\beta \right]^{1/\beta}}$$

The value of Gini coefficient lies between zero and one (0<Gini<1). Gini coefficient equal to zero shows a perfectly equal distribution of income (every person has the same level of income) while equal to one shows a perfectly unequal distribution of income (some persons have more income and some have less income). Value of Gini coefficient close to zero shows more equal distribution of income while close to zero shows unequal distribution.

Choice of different dimensions for multidimensional inequality is a crucial task and can be chose with various justifications as given in the preceding section. Following dimensions are used for multidimensional inequality: Monetary wellbeing (three possible choices for material wellbeing: income, asset ownership and expenditures), educational achievement (three possible choices of indicators for education: an average level of education of an adult household member, the complete year of education of household head, and maximum complete years of education by any member of a household), Health (body mass index (BMI), health risk index, Health risk index combines three sub-indicators: access to safe drinking water, improved toilet and crowding).

4. Results and Discussion:

4.1 Multidimensional poverty

Poverty is not merely the lack of certain level of income/ expenditures. Income or expenditures are not only the determinant of poverty, many factors other than income/expenditures also contribute in poverty like poor health, lack of education , poor housing and low living standard. Poverty is something beyond income deprivation. This study has estimated MPI at national level, provincial level as well as regional level.

4.1.1 Multidimensional Poverty at National Level:

This study applied Alkire and Foster (2007) methodology by using four dimensions; health, education, expenditures and living standard for measuring multidimensional poverty. Figure 1shows the percentage of multidimensional deprived people by using ten indicators under the umbrella of four dimensions and by using equal weights. In 2005-06 51 percent people were multidimensional deprived whereas 49 percent people are multidimensional non-deprived in line with Awan et al. (2012).

Figure 1 Percentage of Multidimensional Deprived Population in 2005-06
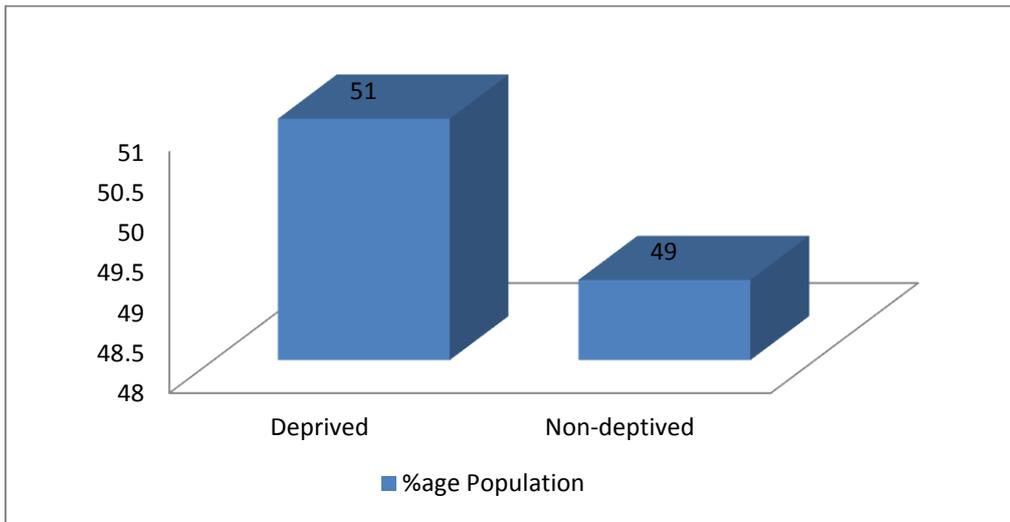
According to multidimensional poverty estimates, 51 percent people were poor.

Figure 2 shows multidimensional poverty estimates for the year of 2010-11. In this time period, multidimensional deprived population was 35.86 percent while the rest were non-deprived. Multidimensional poverty has declined by 15 percent during 2005-06 to 2010-11.

**Figure 2** Percentage of Multidimensional Deprived Population in 2010-11
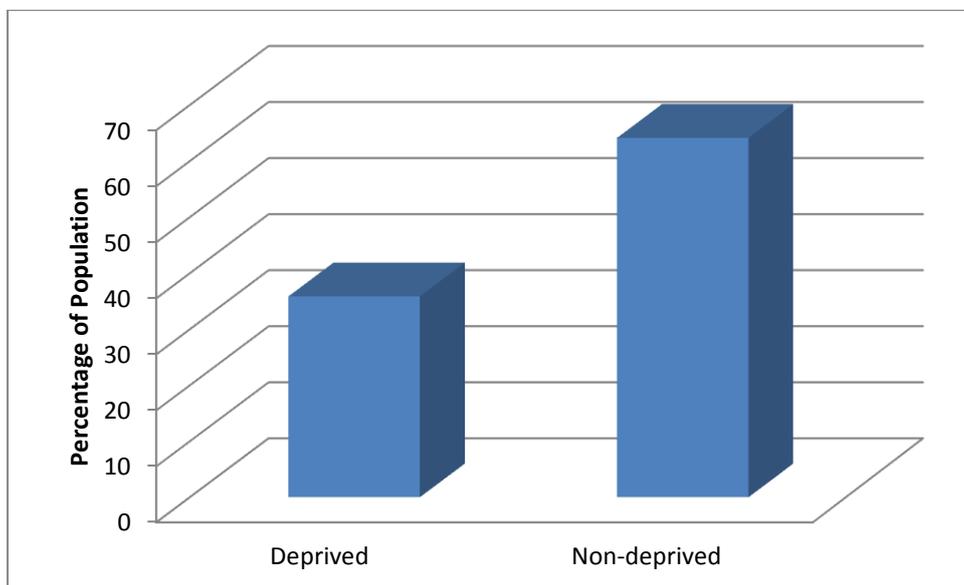
In 2010-11 expenditure poor were 12 percent in Pakistan while multidimensional poor were almost three times greater than expenditure. The reduction rate in multidimensional poverty was 0.05 percent per annum.

4.2 Dimensional Share to overall MPI

Figure 3 illustrates the share of each dimension; expenditures, education, health and living Standard to national multidimensional poverty in 2005-06.

Figure 3 Dimensional Share to National Poverty 2005-06
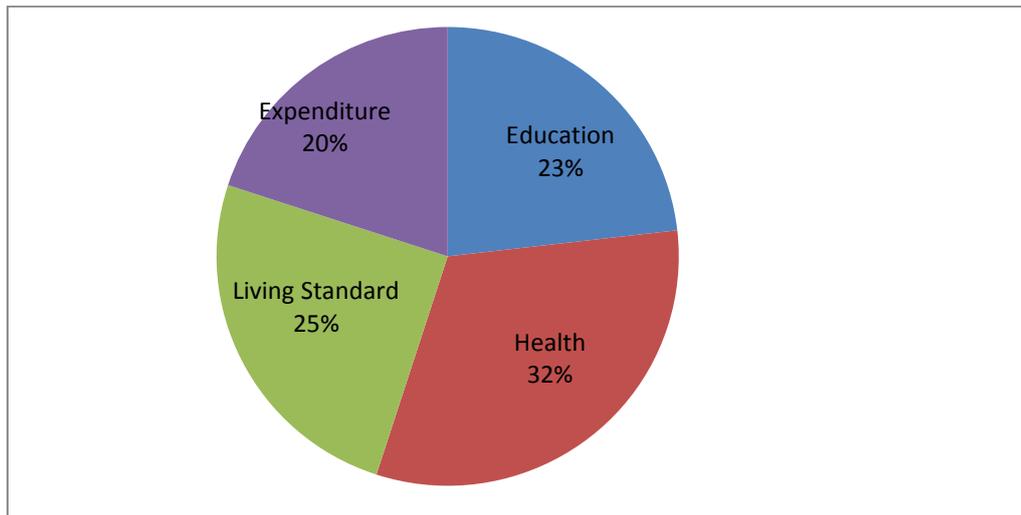
Source: author's own calculations

From Figure it is clear that education[1] contributes 23 percent to overall poverty, Health[2] contributes 32 percent while living Standard[3] contribution is 25 percent and expenditures share to overall poverty is 20 percent. Health depicted higher contribution to overall poverty while expenditures (traditional tool for poverty estimates) have least contribution to overall poverty.

Figure 4 represents the dimensional share to overall poverty in 2010-11. 2010-11. Analysis has the same dimensions; expenditure, health, education and living standard as used in 2005-06 analysis. Moreover all dimensions have same indicators and definitions as used in 2005-06.

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[1]Dimension of education consist of two indicators: enrolment of children aged between 4-9 years and complete years of education.

[2]Health further comprises of two indicators: immunization for measles and post natal care.

[3]Living standard further includes five indicators: toilet facility, access to safe drinking water, electricity, gas and crowding.

**Figure 4** Dimensional Share to National Poverty in 2010-11
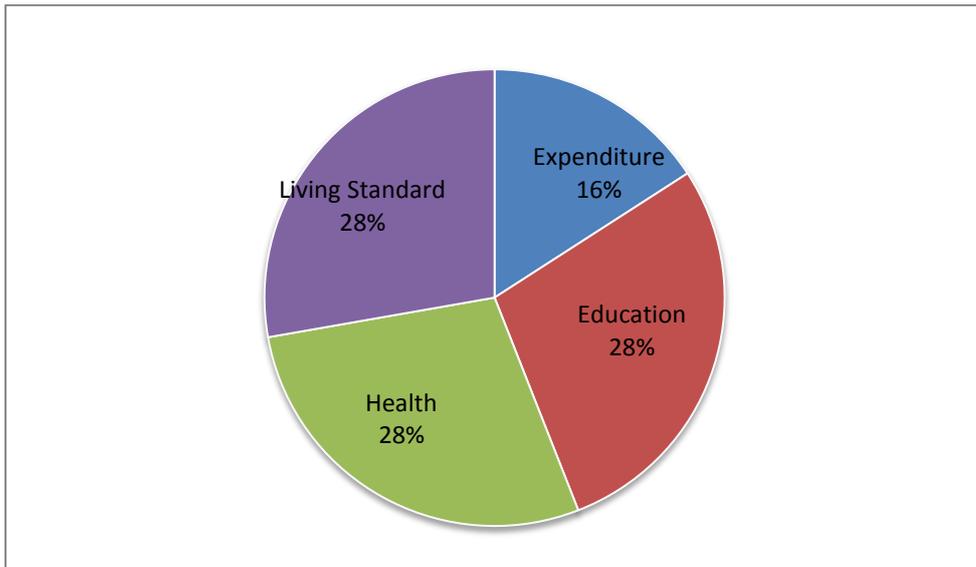
Source: author's own calculations

Above figure shows that all dimensions; health, living standard and health, have almost equal contribution to overall poverty. The traditional tool for poverty measurement has least contribution to overall poverty which is 16 percent and also showed similar scenarios as in 2005-06 data. So, uni-dimensional poverty can't give a real picture of deprivation. It overlooks the other necessary aspects of poverty and to grip the problem of poverty multidimensional analysis is needed. Contribution of health and living standard has increased over time. Thus these findings concluded that to curb multidimensional deprivation resources must be allocated according to the percentage of the share of each dimension to overall poverty.

Dimensional contribution can be more zoomed by considering percentage deprivation in each indicator. Figure 5 gives a more detailed picture of multidimensional poverty through percentage of deprived population in each indicator.

Figure 5 Percentage of Household Deprived in each indicator during 20005-06
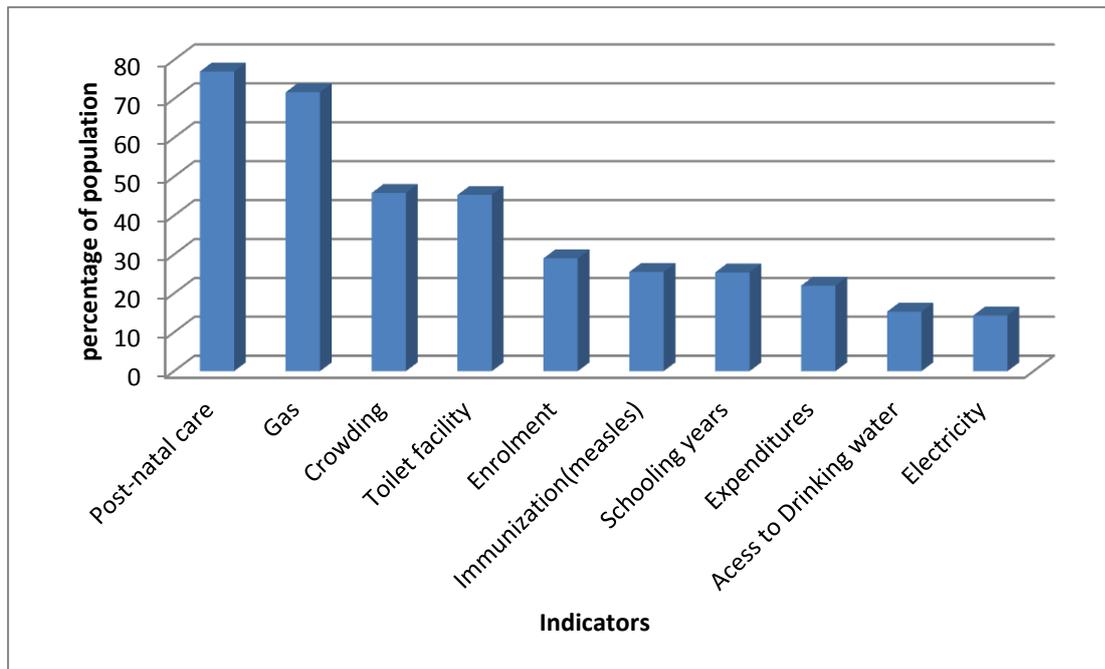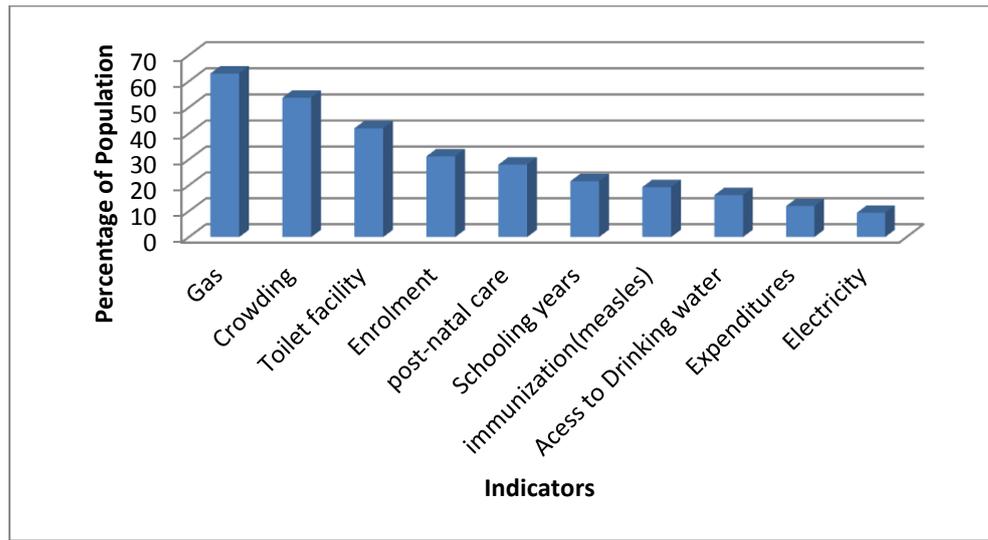

Source: author's own calculations

Above diagram illustrates that during 2005-06, about 77 percent households are deprived in postnatal care, 25.5 percent households in immunization against measles, 71.7 percent households have not a gas connection. It is also depicted that the households have small houses as compared to their family, i.e. in 45.8 percent households more than three persons share a room. Congestion may cause many diseases which spread through respiration. 45.3 percent of households have no access to an improved toilet (this supports the Govt. statistics provided in PSLM report 2005-06). These statistics are close to earlier study (Awan, 2012) i.e. 40.5 percent. About in 29 percent household destitute in enrolment while 25.4 percent households have not even a single person who has completed his primary education. Current enrolment shows that the household's future wellbeing. It will decline if even a single school going age child is not enrolled in school. In Pakistan 15.3 percent households have no access to safe drinking water or it may consume 30 minutes to reach at the source of drinking. The 14.3 percent household has no electricity connection as a lightning source while 22 percent of

the population was living below poverty line (BPL) which supports the National statistics of poverty. It means that 22 percent household don't generate sufficient salary to fulfil their basic needs. Figure 4.8 illustrates the percentage of household deprived in each dimension in 2010-11.

**Figure 6** Percentage of Household Deprived in each indicator during 2010-11

Source: author's own calculations

Figure shows that during 2010-11, about 63 percent households have not gas connection (also confirms govt. statistics of PSLM 2010-11). It is also clear that 53.7 percent households have small houses as compared to their family, i.e. in 53.7 percent households more than three persons share a room. 42 percent of households have no access to an improved toilet and toile facility improved than 2005-06. About in 31.2 percent household destitute in enrolment while 21.6 percent household has not even a single person who has completed his primary education. 28 percent households are deprived in postnatal care (also confirms govt. statistics of PSLM 2010-1), 19.3 percent households are destitute in immunization against measles (confirms govt. statistics of PSLM 2010-1). 16.3 percent households have no access to safe drinking water and this condition is worse than 2005-06. Almost 9.2 percent household has no electricity connection as a lightning source (confirms govt. statistics of PSLM 2010-1). At

the last expenditure dimension has 12 percent household deprived, these are close to national poverty statistics of Government of Pakistan.

Figure 7 Indicator Wise Contribution to Overall Poverty
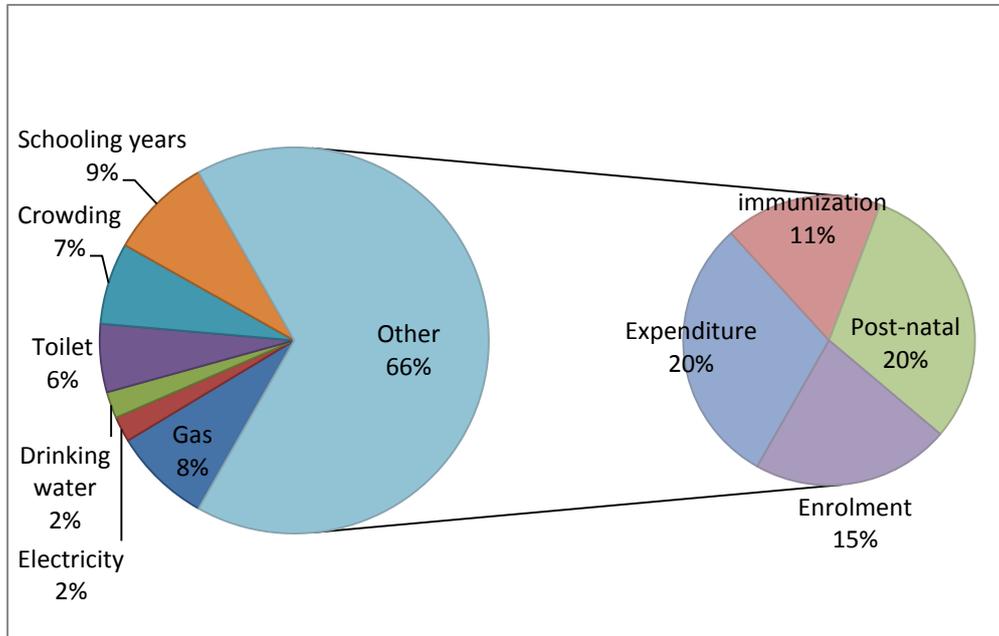
Figure 7 represents that expenditures, post-natal care, enrolment and expenditures contribute 66 percent to overall poverty. It indicated that only focussing on these four dimensions 66 percent of overall multidimensional poverty can be control.

**Figure 8** Indicator Wise Contribution to Overall Poverty 2010-11
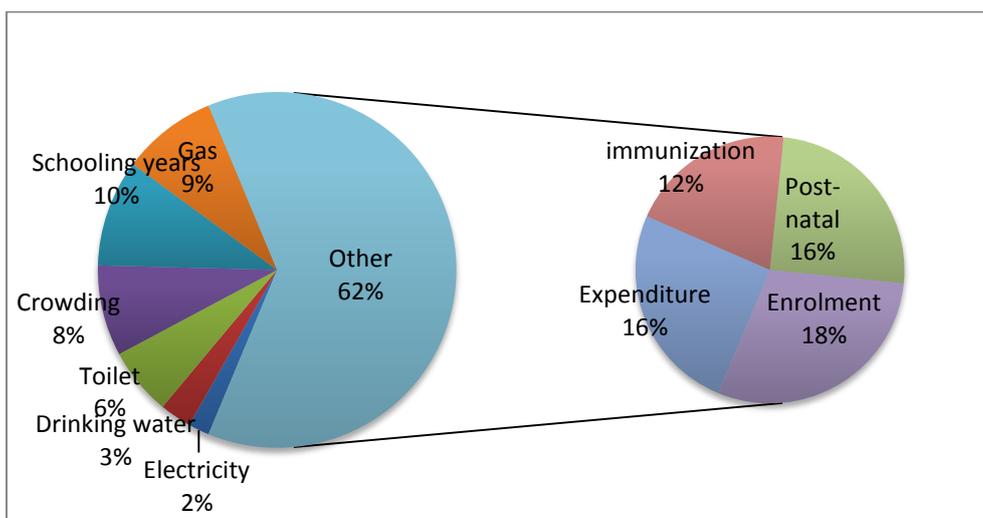
Figure 8 represents that in 2010-11 contribution of these dimensions has declined from 66 percent to 62 percent in overall poverty. It means still these four indicators have major share in overall poverty. Poverty reduction policies must target these indicators for significant reduction of overall poverty.

In multidimensional poverty, poverty cutoff, minimal level of deprivation above which a household is declared as poor is required to be fixed. The aggregate cutoff is just like a poverty line falling lower than or at the poverty line person is considered as poor. Multidimensional poor at different cut-off levels are given in Figure 9. It represents that at poverty cut-off equal to two (k=2), headcount ratio is 92 percent this is a union approach while at k=10 there is no poverty at all, this is the intersection approach. Union approach overestimates while the intersection approach underestimates the poverty.

Figure 9 Headcount at different cut offs in 2005-06
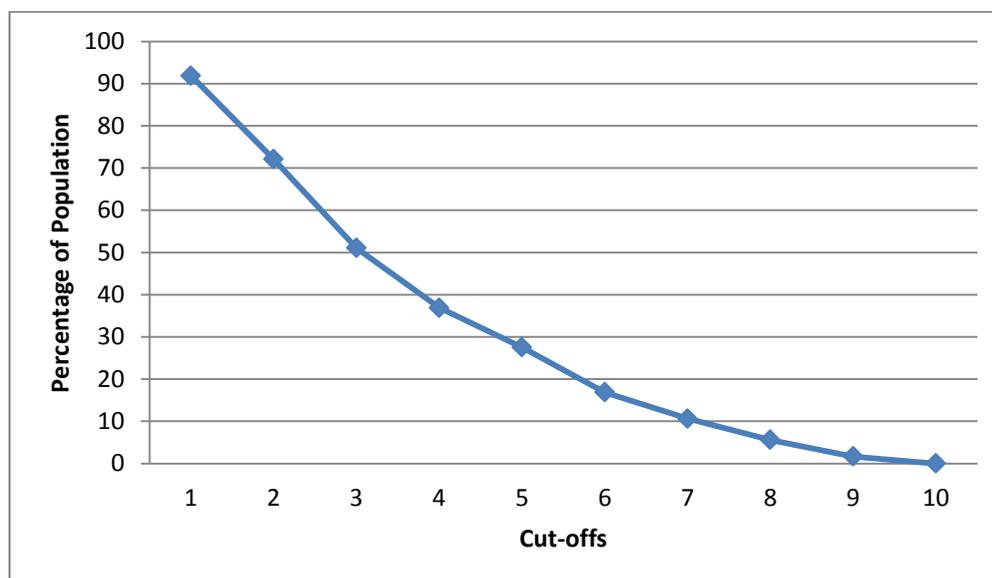
Source: author's own calculations

According to dual cut off approach, poverty cut off "K" must lie between two approaches at "K-2" headcount ratio is 72 percent. As "K" increases headcount ratio decreases such as at k equal to 3, 4, 5, 6, 7 and 8 headcount ratio is 51 percent, 37 percent, 28 percent, 17 percent, 11 percent and 2 percent respectively. Figure 10 shows headcount ratio for 2010-11.

Figure 10 Headcount at different cut offs in 2010-11
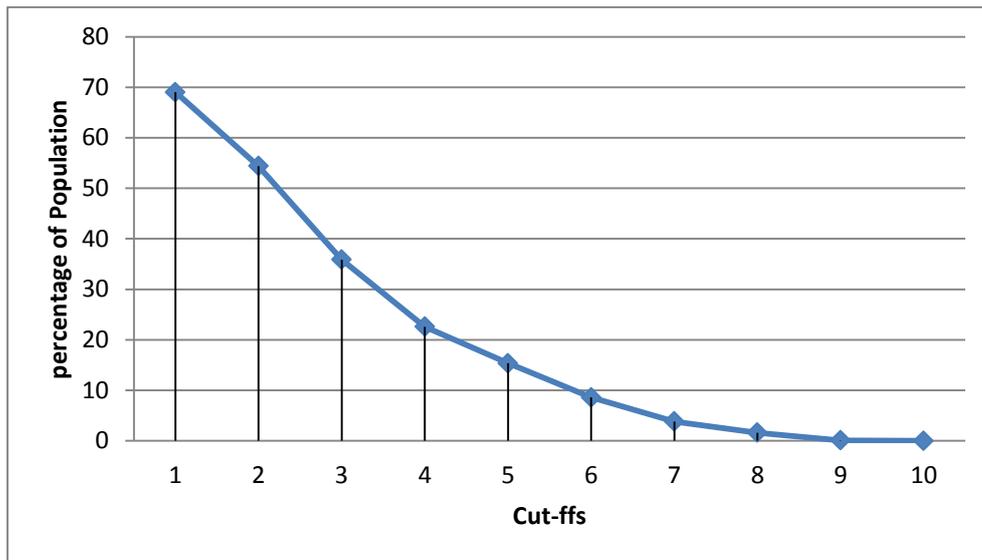
Source: author's own calculations

According to union approach headcount ratio is 69 percent whereas intersection approach shows 0 percent headcount ratio for 2010-11. Regarding dual cut off approach, poverty cut off "K" must be lie between two approaches. at "K-2" headcount ratio is 72 percent. As "K" increases headcount ratio decreases such as at k equal to 2,3, 4, 5, 6, 7 and 8 headcount ration is 54 percent, 36 percent, 23 percent, 15 percent, 9 percent, 4 percent and 2 percent respectively. The estimates of headcount ratio illustrates that multidimensional poverty has decline at all cut offs during 2005-06 to 2010-11.

Alkire and Foster methodology take into consideration the intensity of deprivation also. Table 1 shows the intensity of deprivation as well as adjusted multidimensional headcount ratio for the year 2005-06.

Table 1 *Adjusted Headcount Ratio at different Cut Offs (2005-06)*

K	H	A	$M_o = H \times A$
1	0.9186	0.4008	0.3682
2	0.7207	0.4681	0.3374

3	0.5106	0.5555	0.2836
4	0.3685	0.6271	0.2311
5	0.2753	0.6852	0.1886
6	0.1686	0.7662	0.1292
7	0.106	0.8365	0.0887
8	0.0558	0.8906	0.0497
9	0.0164	0.9664	0.0158
10	0	0	0

Source: author's own calculations

Column 1shows different cut offs (K), column 2 shows headcount ratio (H), column 3 shows intensity of multidimensional deprivation(A) whereas column 4 represents adjusted headcount ratio(M_0) which is the product of the raw headcount ratio and intensity of deprivation. At K=2 adjusted headcount ratio is 0.3682(36.82 percent) and it declines as K increases. Headcount is 91.86 percent and intensity is 40 percent which means that 91.86 percent households are poor in 40 percent dimensions on average. Value of "A" increases as "K" increases.

Table 2 *Adjusted Headcount Ratio at different Cut Offs (2010-11)*

K	H	A	$M_o = H \times A$
1	0.6002	0.3655	0.2523

2	0.5440	0.4207	0.2289
3	0.3586	0.5056	0.1813
4	0.2260	0.5866	0.1326
5	0.1537	0.6453	0.0992
6	0.0860	0.7309	0.0629
7	0.0384	0.8071	0.0310
8	0.0016	0.8816	0.0141
9	0.0010	0.9646	0.0010
10	0	0	0

Source: author's own calculations

Table 2 shows the multidimensional poverty estimates for the year of 2010-11. Multidimensional poverty has declined over time. Here also as "K" increases intensity also increases while the raw headcount ratio increases too. Usually 33.3 percent "K" is used as the poverty line which means that if a household has more than 33.3 percent deprivation then it is declared to be multidimensional poor.

4.3 Results of Inequality in Pakistan

This study analysed inequality in wellbeing in Pakistan from for 2005-06 and 2010-11. Data used in this section also drawn from PSLM conducted by Pakistan Bureau of Statistics. In addition to consumption expenditure, facts of health and education also incorporated. Equivalent per adult expenditures, proposed by the Planning Commission of Pakistan, were

used. In analysis maximum years of education of any of the family member and the Health Risk Index were included.

For computation of Gini coefficient, the researcher decided four fundamental decisions about: standardization of each indicator, the degree of substitution (value of β), weights and inequality aversion parameter (value of δ). Each outcome was divided with its respective mean for standardization, the value of β was restricted to zero because ratio-scale invariance property required Cobb–Douglas function used and Value of parameter infers value judgment about indicators and it is difficult to say which indicator is most important in human welfare so equal weights are employed in analysis[4]. Koen (2009) cited that Human Development Index (HDI) used β=1, which means all indicators are perfectly substitutable, but it is contradictory for human development (Human Development report UNDP, 2005, p.22).

Standard Gini coefficient possessed the value of δ equal to 2. This study computed Gini Coefficient for two values of delta one value was equal to 2 and other was 5 which represent the higher weight for bottom distribution. Results are given below;

Table 3 Gini Coefficient in 2005-06 and 2010-11

Gini coefficient	Delta=2		Delta=5	
	2005-06	2010-11	2005-06	2010-11
Expenditure	.32	.29	.5	.45
Health risk index	.19	.21	.34	.38

[4]HDI also used equal weights.

Max. education	.34	.33	.69	.68
Multidimensional	.35	.34	.61	.61
Welfare Index	.64	.65	.38	.39

Source: author's own calculations

Table 3 illustrated consumption inequality in different wellbeing dimensions in Pakistan 2005-06 to 2010-11. Value of Gini-coefficient ranges from 0 to 1 where zero means perfect equality and 1 means perfect inequality. Gini index shows the percentage of inequality prevailing in a country. The table portrayed that consumption expenditure inequality has declining trend from 0.32 to 0.29 for delta equal to two and more sever when value of $\delta=5$ but inequality has decreased for both values of delta. Health risk inequality has increased from 0.19 to 0.21for delta equal to 2 and also increased 0.34 to 0.38 for delta equal to 5. Education inequality declined for delta equal to 2 and remained same when value of delta is 5. Multidimensional inequality showed slightly decline in dispersion when the delta is 2 but showed no significant reduction in dispersion for delta equal to 5. Welfare index showed welfare has only 1 percent reduction for both values of delta and welfare has declined about 26 percent with the increase in value of delta from 2 to 5. Trend of results was in line with Koen (2012). Upshot, results showed that from 2005-06 consumption inequality has declined about 3 percent while multidimensional inequality has declined only 1 percent.

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