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Modelling Foreign Exchange Risk in a Managed Float Regime: Challenges for Pakistan

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We study the implications of the exchange rate regimes (managed vs. floating) for implementing risk assessment models for Pakistan which seems to manage its currency mainly against the US dollar, but to a lesser extent against other hard currencies. We test five variations of the Value-a-Risk (VaR) model, including models based on the Extreme Value Theory (EVT). Our results indicate that these models do not perform as well for the currency pairs with the managed float (USD/PKR and JPY/PKR). It implies that the managed float regime imposes additional risk and cost on the economic agents. The findings of this paper provide additional support for following a free float policy. Our findings also underscore the importance of correctly specifying the VaR model in a dynamic framework.

Keywords: Value at Risk, Risk Management, Managed Float, Extreme Value Theory

1. INTRODUCTION

Since the abandonment of fixed rate regimes with the Smithsonian Agreement in 1973, the dominant view among economists is that floating exchange rates, wherein a currency's value is allowed to fluctuate according to the foreign exchange market, are preferable to the fixed exchange (FX) rates. However, fixed exchange rates may be preferred by economic policy-makers as they may bring in greater short-term stability, while free floating exchange rates increase foreign exchange volatility. This is an important consideration especially for the emerging economies which typically face three conditions: (i) their liabilities are denominated in foreign currencies while their assets are in the local currency, (ii) their financial systems are fragile, and (iii) bank and corporate balance sheets are vulnerable to exchange rate deterioration. For such reason emerging countries exhibit a *fear of floating* [Calvo and Reinhart (2002)]. Therefore, though officially following the free-float regime, a central bank will occasionally intervene in the currency market to stabilise its value, and thus manage the float. Consequently, the number of countries that manage the float increased significantly during the 1990s, and currently the majority of the world's currencies are on managed float, aka *dirty float*.

However, the environment of managed float regimes has rendered the management of foreign exchange risk, using models such as the Value at Risk (VaR), more

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challenging for a number of reasons.[1] Firstly, "… there are substantial and systematic differences in the behaviour of real exchange rates under these two nominal exchange rate regimes;" Mussa (1986). Genberg and Swoboda (2004) further document that the "properties of the frequency distribution of changes in exchange rates are different in countries that announce that they are following a fixed exchange rate regime compared to countries that are officially floating". More interestingly, the authors note that the properties of the tails of the distributions are different for the two foreign exchange regimes, i.e., the *de jure* fixed category contain a higher frequency of large exchange rate changes (of either sign) compared to the *de jure* float category. There is also a growing divergence between the *de facto* and *de jure* exchange rate regimes followed by the central banks; most countries following a *dirty float*, which makes it more difficult to implement risk models.

Second, as Engel and Hakkio (1993) explain, the system of fixed but adjustable rates introduces a new kind of volatility: volatility caused by the expectations of exchange rate realignments. By eliminating the market's uncertainty about the future exchange rate, a system of absolutely fixed exchange rates reduces *normal* volatility. However, when the rates are fixed but adjustable, the market knows that realignment may occur, the speculation around the magnitude and timing of the realignment will exacerbate exchange rate volatility. Therefore, between realignments, exchange rate volatility will tend to be within normal limits, but around the time of realignments it can be extreme. If the equilibrium rate continues to trend upward or downward, then the incidence of realignment increases, and with it the incidence of extreme volatility also increases.

Third, at a more fundamental level, the empirical return distributions of financial assets are found to be fat tailed and skewed in contrast to the Normal Distribution as assumed in the theoretical models. An extensive literature in finance (e.g., Nassim Taleb's *The Black Swan*) underscores the importance of rare events in risk management which materialise as large positive or negative investment returns, a stock market crash, major defaults, or the collapse of risky asset prices. Therefore, in order to model foreign exchange risk we need to address the issue of extreme observations or heavy tails of distributions. In response, VaR risk measures based on the *Extreme Value Theory* (EVT) have been developed which allows us to model the tails of distributions and to estimate the probabilities of the extreme movements that can be expected in financial markets. The basic idea behind EVT is that in applications where one is concerned about the risk of extreme loss, it is more appropriate to separately model the tails of the return distribution.

Our objective in this paper is to examine the question whether the empirical exchange rate distributions in the managed float regime (i.e., against US Dollar in Pakistan, or the Yen in Japan) would be more or less amenable to risk modelling, than for the currencies (i.e., Pound, Euro, and Euro) not so managed. Our research addresses this question by comparing various Value-at-Risk models applied to the four exchange rates, employing back-testing techniques and examining the incidence of actual losses exceeding those predicted by the risk models. Although a number of research papers

[1]Value at Risk (VaR) is the most widely used measure of market risk, which is defined as the maximum possible loss to the value of financial assets with a given probability over a certain time horizon.

address the merits and demerits of the two exchange rate regimes, the choice of the regime also affects the risk management practices. This aspect has not received as much attention in prior research.

Pakistan offers an instructive case study in two aspects. First, because of the particular foreign exchange regime followed by the country; apparently, in Pakistan the US Dollar (USD) is the main currency being managed, while the other currencies are not being so managed, and the exchange rates are being dictated by the cross-rates. Pakistan has pursued different exchange rate regimes in its history spreading over 70 years.[2] Following the worldwide trend of deregulation and liberalisation, Pakistan opted out of the fixed exchange rate regime and floated the rupee against a basket of sixteen currencies under a managed exchange rate regime in 1982. After a short period (1998-2000) of experimentation with two tier system and dirty float, in July 2000, the SBP officially moved away from the managed exchange rate to a floating exchange rate regime. However, Pakistan is categorised as a managed floater per its official pronouncements. IMF's de facto classification of exchange rate regimes, as of July 31, 2006, notes that, "the regime operating *de facto* in the country is different from its *de jure* regime," and categorises Pakistan as following "other conventional fixed peg arrangements". A study by Rajan (2012) examining the exchange rate regimes in Asian countries over 1999-2009 period finds that, "Pakistan seems to operate rather *ad hoc* adjustable pegs." It, however, finds insufficient evidence for the existence of any systematic exchange rate fixity, but notes a high degree of influence of the US dollar and negligible influence of the other currencies for Pakistan, suggesting that the country manage its currencies against the US dollar. Therefore, we can compare the performance of the risk models under the two regimes with reference to different foreign exchange rate pairs.

Second, the country in the recent past has suffered a series of economic shocks ranging from an ongoing incidence of terrorism to natural floods. Worsening economic conditions in the country, deteriorating law and order situation, energy crisis and terrorism, have led to steady depreciation in the value of rupee. Unsettled political issues, uncertainties surrounding the flow of foreign aid, combined with weak macro-economic management have not provided an ideal set of circumstances for the execution of a managed float regime. During last decade, the financial markets have experienced high volatility and incidence of extreme returns, in particular, following the global financial crisis of 2007-09 (GFC) Pakistani Rupee depreciated by 23 percent against US Dollar. Thus, the country provides us with a rich dataset of extreme observations; a large number of extreme observations are needed for evaluating risk models based on the Extreme Value Theory.

2. EXCHANGE RATE RISK MODELS

The use of VaR model has become the standard practice with the introduction of J.P. Morgan's RiskMetrics™ in 1994 and the Basle II agreement in 2004 which is based on the empirical distributions of short-term asset returns. Boothe and Classman (1987) provide a comprehensive survey of the theoretical and empirical work on the unconditional distribution of foreign exchange rate returns. There is extensive evidence that the empirical distributions of

[2]See Janjua (2007) for details on the history of exchange rate regimes in Pakistan.

foreign currency returns are fat-tailed. Koedijk, *et al.* (1990) based on their analysis of EMS rates, suggested using Extreme Value Theory to model exchange rate return. Therefore, an integration of the EVT with the VaR models is a logical extension. Yet, in practice EVT based risk models have not been widely adopted.

With respect to the emerging markets various academic studies establish the applicability of the VaR models. For example, Al-Janabi (2006), demonstrates the management of trading risk exposure of foreign-exchange securities in the Moroccan market. Hooy, Tan and Nassir (2004) document significant impact of exchange-rate exposure on the Malaysian banks; they find that the severity of exchange-rate risk remained constant before and after financial crisis. Nath and Reddy (2003) apply three different VaR models to the FOREX market in India including a tail-index model based on EVT. They, however, find that most of the models are failing in a rolling window framework, while the full sample data overestimates the VaR. Ajili (2008) assesses the exchange risk associated to the Tunisian public debt portfolio using delta-normal VaR application, and demonstrate that the VaR approach can be used for a small developing economy. Mapa, *et al.* (2010) propose a method of formulating VaR using the Generalised Pareto Distribution (GPD) with time-varying parameters. They test the proposed model for the Philippine Peso-US Dollar exchange rate over 1997-2009 and show that the models were better-performing in predicting losses from exchange rate risk, and have potential as part of the VaR modelling. In a recent paper Wang, *et al*. (2010) applied EVT to estimate the tails of the Chinese Yuan (CNY) exchange rates against major currencies and measured risk using VaR and Expected Shortfall techniques. Similarly, de Jesus (2013) measured Value-at-Risk of peso/dollar exchange rates using EVT. Purevsuren (2010) illustrates how EVT can be used to model tail-related risk measures and tests the methods using out-of-sample analysis for a portfolio consisting in four Mongolian foreign exchange rates. A study related to measuring risk of FOREX market in Pakistan is by Akbar and Chauveau (2009). The authors apply historical simulation, Monte Carlo simulation and delta-normal VaR technique to assess exchange rate risk exposure of public debt portfolio of Pakistan.

While these studies have examined how far the VaR models can be efficacious in managing the foreign exchange risk, we focus on the question whether these models perform better or worse for the Pakistan Rupee against foreign currencies when either of the currency in the pair is under managed float, USD or JPY, versus other currencies (GBP or EUR) which as not being so managed.

3. MODELS, DATA AND METHODOLOGY

The study evaluates exchange rate risk of Pakistani Rupee (PKR) based on the Value-at-Risk (VAR) models against four major trading currencies i.e., US Dollar (USD), European Euro (EUR), British Pound (GBP) and Japanese Yen (JPY) for the period January 1999 to August 2017. The four currencies are selected on the basis of their long-term predominance in foreign exchange transactions (almost accounting for 95 percent of both payments and receipts).The sample period of about 19 years consists of 4822 to 4841 daily observations for the four exchange rates. The time span starts after the Asian currency crisis of late 1990's and just before the country moved to adopt the current foreign exchange regime. The "returns" are measured as the first log differences of the exchange rate series i.e.:

$R_{t,EUR} = \ln((EUR/PKR)_t / (EUR/PKR)_{t-1})$
$R_{t,USD} = \ln((USD/PKR)_t / (USD/PKR)_{t-1})$
$R_{t,GBP} = \ln((GBP/PKR)_t / (GBP/PKR)_{t-1})$
$R_{t,JPY} = \ln((JPY/PKR)_t / (JPY/PKR)_{t-1})$

The purpose of converting exchange rates into geometric returns is to achieve stationarity which is confirmed by the results of the Ducky Fuller tests as reported in Table 1. It should be noted that US Dollar and the British Pound are classified as *free-float* currencies while the Japanese Yen is considered being a managed float currency.

Table 1

Augmented Ducky Fuller Unit Root Test

R_t	EUR	USD	GBP	JPY
t-Statistic	–68.687	–61.057	–70.952	–53.547
Probability	0.0001***	0.0001***	0.0001***	0.0001***

The null hypothesis assumes that the series has a unit root and *** indicates rejection of null hypothesis at 1 percent level of significance.

3.1. Value at Risk and Conditional Volatility

Value at Risk (VaR) is a high quantile (typically the 95th or 99th percentile) of the distribution of returns and provides an upper bound on tails of returns distribution with a specified probability. However, classical VaR measures based on the assumption of normal distribution of the financial asset underestimate risk as the actual return distributions exhibit heavier tails. One alternative to deal with the non-normality of the financial asset distributions has been to employ historical simulation methodology which does not make any distributional assumptions, and the risk measures are calculated directly from the past observed returns. However, the historical approach sill assumes that the distribution of past returns will be stable in the future. Another approach is to use Extreme Value Theory (EVT) to construct models which account for such thick tails as are empirically observed.

Although EVT is an appropriate approach for modelling the tail behaviour of stock returns, the assumption of constant volatility is contradicted by the well documented phenomenon of volatility clustering i.e., large changes in assets values are followed by large changes in either direction. Hence, a VaR calculated in a period of relative calm may seriously underestimate risk in a period of higher volatility.[3] The time varying volatility was first modelled as a ARCH (q) process [Bollerslev, *et al.* (1992)] which relates time t volatility to past squared returns up to q lags. The ARCH (q) model was expanded to include dependencies up to p lags of the past volatility. The expanded models, GARCH (p,q) have become the standard methodology to incorporate dynamic volatility in financial time series; see Poon and Granger (2003). Similarly the auto-correlation of returns is significant in many situations and there is also a need to incorporate the ARMA (m,n) structure in the model.

[3]See Hull and White (1998) Acknowledging the need to incorporate time varying volatility VaR models employ various dynamic risk measures such as the Random Walk model, the GARCH, and the Exponentially Weighted Moving Average (EWMA).The Riskmetrics model uses the EWMA.

Our preliminary checks on the data lead us to identify different dynamic processes for the four currencies (see next section for details). The choice of the models is based on the principle of parsimony, and is supported by an examination of the standardised residuals extracted from the models.

Currency	*Model Specification*
EUR	AR(1)-GARCH (1,1)
USD	AR(2)-GARCH (1,1)
GBP	AR(1)-GARCH(1,2)
JPY	AR(1)-GARCH(2,1)

3.2. Applying Extreme Value Theory

After applying the appropriate the GARCH(p,q) models to the four series, the residuals from each model are extracted. The next step is to model the tails of the innovation distribution Z_t of these residuals, using the Extreme Value Theory, as explained in the Appendix. The estimation of the GPD parameters, ξ and β is made using the method of maximum likelihood. Finally the estimated dynamic or conditional VaR equation (see Appendix) is: $\hat{x}_q^t = \widehat{\mu_{t+1}} + \hat{z}_q \widehat{\sigma_{t+1}}$. We run five different VaR models as have been suggested in the literature, as follows:

(i) Conditional EVT, a VaR model based on EVT and incorporating GARCH) effects;
(ii) Conditional Normal, a VaR model based on Normal Distribution, and incorporating GARCH effects;
(iii) Conditional-t, a VaR model in which in which GARCH effects are incorporated and innovations are assumed to have a Student's-t Distribution;
(iv) Unconditional EVT method, a VaR model based on Extreme Value Theory but GARCH effects are not incorporated;
(v) Unconditional Normal, a VaR model in which data are assumed to be normally distributed and GARCH effects are not incorporated.

The first three models incorporate the dynamics of volatility, the GARCH effect. Models (i) and (iv) make use of the Extreme Value Theory. Model (iii) offers an alternative to the EVT approach by employing the t-distribution when innovations may have a leptokurtic distribution. Thus, the five models allow us to draw comparisons as to the efficacy of different models for risk assessment and management.

3.3. Back-testing

After applying the five VaR models, we back-test the models on historical series of log-returns $\{x_1, x_2, \ldots, x_n\}$.We calculate $\hat{x}_q^t$ on day t in the set $T=\{m,m+1,\ldots,n-1\}$ using a time window of m days each time. Similar to McNeil and Frey (2000), we set m=1000, but we consider 50 extreme observations from the upper tail of the innovation distribution i.e., we fix k=50 each time. On each day $t \in T$, we fit a new GARCH(p,q) model for the four foreign exchange returns series and determine a new set of GPD parameter estimates. We compare $\hat{x}_q^t$ with x_{t+1} for three quantile levels, $\in\{0.95, 0.975, 0.99\}$, for the four exchange rate series. A violation is said to occur whenever $x_{t+1} > \hat{x}_q^t$.

We then apply a one-sided binomial test based on the number of violations for assessing the model's adequacy.

4. EMPIRICAL RESULTS AND DISCUSSION

Table 1 reveals the descriptive statistics of return series. The mean for all series is positive, which reflects over the period devaluation of PKR with respect to the hard currencies. PKR's devalued relatively more against the Yen, and to a lesser degree against the British Pound. Note that, since we are stating the exchange rate as rupees per unit of foreign currency, a positive change represents a loss in the value of rupee. The mean and the maximum appreciation and depreciation of the Pakistani Rupee on day-to-day basis is almost similar against all four currencies. The maximum one day fall of Rupee against Euro, Pound and Dollar is around 4 percent whereas against Yen the maximum one day depreciation is 8 percent.

It is notable that daily variation measured by the standard deviation of the daily exchange rate returns is the least against Dollar; it is less than half of the standard deviations for the other currencies. The returns series in all cases have excess kurtosis (measure greater than 3) which implies the presence of outliers in daily exchange rate returns. As noted, the minimum and the maximum values are very large relative to the mean, which also indicate heavy incidence of extreme returns. There is also a considerable difference in the skewness measures. In particular, comparing the kurtosis statistics, we notice that the tails of USD/PKR return distribution are remarkably heavier than the tails of the other currency return distribution. The highest value of kurtosis in case of Dollar against Rupee indicates the frequent presence of abnormal daily exchange rate returns. The exchange rate returns in all four cases do not follow the Normal Distribution evident by the significant value of Jarque-Bera statistic; it is remarkably so in case of Dollar. These results support our argument that the USD is the main object of a managed float policy, and strengthen our case for separately modeling the tails of the distribution for risk assessment using EVT.

Table 2

Summary Statistics

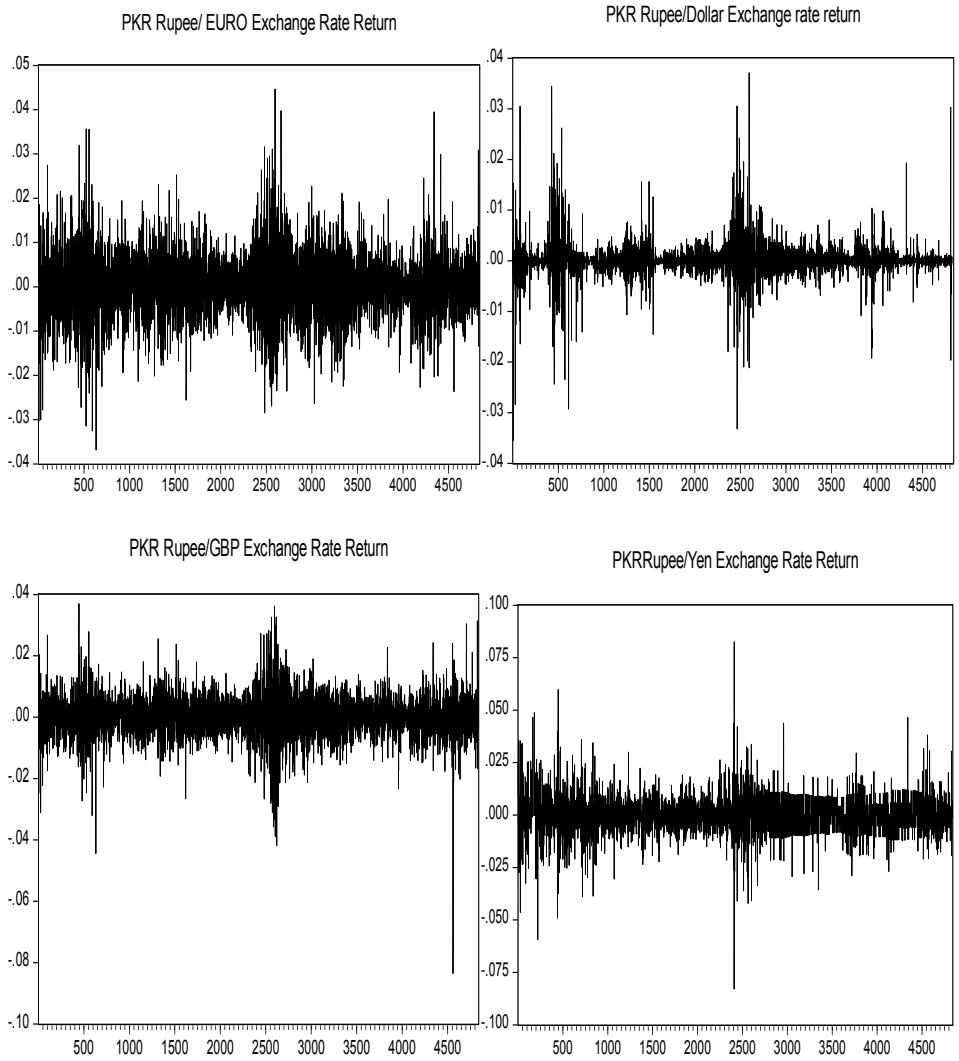
Series	NOB	Mean	Max	Min	Std.Dev.	Skewness	Kurtosis	Jarque-Bera
$\boldsymbol{R_{t,EUR}}$	4840	0.00015	0.0447	-0.0369	0.0071	0.1398	5.5139	1,290.3***
$\boldsymbol{R_{t,USD}}$	4822	0.00015	0.0371	-0.0354	0.0033	0.3718	30.9164	155,690.8***
$\boldsymbol{R_{t,GBP}}$	4840	0.00010	0.0369	-0.0835	0.0067	-0.5541	11.5040	14,832.0***
$\boldsymbol{R_{t,JPY)}}$	4841	0.00016	0.0826	-0.0829	0.0081	0.1300	12.4221	17,920.6***

Note: The null hypothesis of Jarque-Bera test statistic assumes that series follows a normal distribution. *** indicates the rejection of null hypothesis at 1 percent level of significance. We use EVIEWS 5.0 and R 2.15.1 for the analysis.

The next step is to estimate the dynamics of conditional mean and volatility of both series, as per models laid out in the previous section. Figure 1 shows the daily returns for the four exchange rate return series. The graph indicates that large changes tend to be followed by large changes of either sign and small changes tend to be followed by small changes. It implies that returns are not independent and identically distributed,

and the volatility clustering phenomenon is present in the data, which is also verfied by the correlogram of squared returns (not shown here).This suggests that GARCH models need to be employed to incorporate dynamic volitality.

Fig. 1. The Plots a-d Show Exchange Rate Return Series for EUR, USD, GBP and JPY respectively.



As noted above our preliminary checks on the data lead us to employ AR(1)-GARCH (1,1) model for EUR and AR(2)-GARCH (1,1) for USD, but AR(1)-GARCH(2,1) for JPY, and AR(1)-GARCH(1,2) models for GBP exchange rate series. The models are fitted using maximum likelihood method. The estimates of the models are given in the following Table 3.

Table 3

GARCH Estimation Results

Dependent Variable		$R_{t,EUR}$	$R_{t,USD}$	$R_{t,GBP}$	$R_{t,JPY}$
			Mean Equation		
Average Return	μ	0.00017	6.03E-05**	0.00013	6.70E-05
		(0.0617)	(0.0333)	(0.0819)	(0.4690)
lag 1	φ_1	-0.05996	-0.2052	-0.0499	-0.0840
		(0.0000)	(0.0000)	(0.0007)	(0.0000)
lag 2	φ_2		0.0249		
			(0.000)		
			Variance Equation		
Constant	ω	2.21E-07***	3.13E-07***	7.15E-07***	4.89E-07***
		(0.0000)	(0.0000)	(0.0000)	(0.0000)
ARCH Effect	a_1	0.0298***	0.1538***	0.0714***	0.1079***
		(0.0000)	(0.0000)	(0.0000)	(0.0000)
	a_2				-0.07586***
					(0.0000)
GARCH Effect	b_1	0.9655***	0.8184***	0.4574***	0.9605***
		(0.0000)	(0.0000)	(0.0000)	(0.0000)
	b_2			0.4551***	
				(0.0000)	
Durbin-Watson Stat		1.9878	1.8172	1.9577	2.0074

The p-values are given in parenthesis; ** indicates the significance at 5 percent level of significance and *** indicates the significance at 1 percent level of significance respectively.

From Table 3 comparing the volatility dynamics of four exchange rate returns, the results implies that the ambient volatility is the highest in case of Pound against the Pakistani Rupee, and the least in case of Euro as indicated by the estimated constant. The dependence of average returns on its immediate past is highly significant in all cases indicated by p-value <0.001. However, the dependence of average daily exchange rate on last day return is negative in all cases. The significant coefficient of AR(2) in case of Dollar indicates that the mean dependence is highest (also justified by the magnitude of AR(1) coefficient) and least in case of the GBP exchange rate return series. The significant value of ARCH effect indicates that the impact of previous shocks on current volatility of exchange rate returns for all four currencies is prominent. In all cases, the (combined) large values of estimated GARCH coefficients (>0.80) indicate the persistence of volatility or in other words a change in volatility affects future volatilities for a long period of time. The effect is the highest in case of EUR/PKR return and the least for USD/PKR. In the conditional variance equation, GARCH effect estimated by (b_1+b_2) is greater than the sum of ARCH coefficients (a_1 + a_2) which explains that the volatility in exchange rate returns depends on its past longer than one period. We next run ARCH-LM test for standardised residuals (Res_t) extracted from the fitted models. The results implies that the extracted residuals are independent identically distributed (*iid*), as indicated by the insignificant p-values in all cases. The results are reported in the Table 4.

Table 4

ARCH LM Residual Test

	$R_{t,EUR}$	$R_{t,USD}$	$R_{t,GBP}$	$R_{t,JPY}$
F-Statistic	2.9295	0.6792	2.7509	0.0713
p-value	0.0870	0.4099	0.0973	0.7895

The test assumes the null hypothesis that the residuals extracted from the fitted models are independent identically distributed.

The Q-Q plots against normal distribution of the residuals series for EUR, USD, GBP and JPY respectively are placed in Appendix A, which indicate the departure from normality and heavy tails for the residual series extracted from fitted model in all four cases.

Following the approach suggested by McNeil and Frey (2000), we apply Extreme Value Theory to model right tail of the standardised residuals extracted from a GARCH model. We consider Peak-Over-Threshold method using the Generalised Pareto Distribution for tail estimation. We consider 95th percentile as the threshold for right tail of standardised residual series in each case. The choice is based on the mean excess (ME) plots placed in the Appendix. The estimates of shape and scale parameters are provide in Table 5. The positive values of estimated shape parameter (ξ) indicate that all the residual series possess heavy tails. The Excess distribution plot (given in the Appendix) indicates that the fitted model is tenable in all cases.

Table 5

Parameter Estimates

	$\boldsymbol{Res_{t,EUR}}$	$\boldsymbol{Res_{t,USD}}$	$\boldsymbol{Res_{t,GBP}}$	$\boldsymbol{Res_{t,JPY}}$
Threshold	1.9714	1.9441	1.9266	2.2833
$\boldsymbol{N_u}$	122	115	124	104
ξ	0.0917	0.4226	0.2458	0.2132
β	0.5765	0.6630	0.4596	0.62931

Next we consider the performance of our suggested model for well-known risk measure known as value-at-risk. We back-test the value-at-risk statistic at 95 percent, 97.5 percent and 99 percent confidence level on historical log-returns, $\{x_1, x_2, \ldots, x_n\}$, for the four series.

Table 6

Back Testing Results for Number of Violations

Length of Test	$R_{t,EUR}$ 3840	$R_{t,USD}$ 3822	$R_{t,GBP}$ 3840	$R_{t,JPY}$ 3840
0.95 Quantile				
# Expected Violations	192	191	192	192
I. Conditional EVT	201 (0.26)	167 (0.04)**	194 (0.45)	183 (0.26)
II. Conditional Normal	172 (0.07)	148 (0.00)**	197 (0.37)	115 (0.00)***
III. Conditional t	179 (0.18)	265 (0.00)***	221 (0.02)**	140 (0.00)***
IV. Unconditional EVT	181 (0.22)	176 (0.14)	220 (0.02)**	241 (0.00)***
V. Unconditional Normal	165 (0.03)**	127 (0.00)***	174 (0.09)	168 (0.04)**
0.975 Quantile				
# Expected Violations	96	95	96	96
I. Conditional EVT	99 (0.39)	89 (0.27)	105 (0.19)	96 (1.00)
II. Conditional Normal	100 (0.35)	99 (0.37)	108 (0.12)	79 (0.04)**
III. Conditional t	88 (0.22)	132 (0.00)***	100 (0.35)	49 (0.00)***
IV. Unconditional EVT	78 (0.03)**	80 (0.06)	100 (0.35)	97 (0.47)
V. Unconditional Normal	93 (0.40)	95 (1.00)	100 (0.35)	105 (0.19)
0.99 Quantile				
# Expected Violations	38	38	38	38
I. Conditional EVT	36 (0.40)	34 (0.28)	35 (0.33)	33 (0.22)
II. Conditional Normal	48 (0.07)	64 (0.00)***	52 (0.02)***	35 (0.33)
III. Conditional t	29 (0.07)	50 (0.04)**	36 (0.39)	10 (0.00)***
IV. Unconditional EVT	36 (0.40)	40 (0.41)	42 (0.30)	41 (0.36)
V. Unconditional Normal	57 (0.00)***	69 (0.00)***	52 (0.02)***	70 (0.00)***

and * indicates the significance of a binomial test at 5 percent and 1 percent level of significance respectively. The one-sided binomial test the null hypothesis with alternative that method systematically underestimates/overestimates the conditional quantile.

Table 6 reports the back testing results and provides theoretically expected number of violations and the observed number of violations using the five different VaR models as explained in the previous section. Whether the observed no of violations is significantly different than expected is measured by the binomial test and the p-values are reported in parenthesis. We consider any outcome where the observed number is different than the expected at a 5 percent or lower level of significance as a failure of the risk model.

We find that the Conditional EVT or the dynamic GARCH-EVT model correctly estimates the conditional quantiles in all cases except one, since the p-value is insignificant at all levels; the method fails only in case of USD/PKR exchange rate return at 95 percent confidence level but still provides accurate results at higher levels of confidence, which indicates that the validity of method holds. Unconditional Normal fails in majority of the cases, seven out of the twelve total cases. It fails especially at 99 percent confidence level. The performance of the unconditional (static) EVT model at higher quantile levels seems satisfactory, since it fails in only three cases out of twelve. Surprisingly, the Conditional–t (or the Dynamic) model does not perform appropriately in most of the cases; it fails in seven out of twelve cases. Conditional-Normal performs well in five out of the twelve cases. Overall, the EVT-based VaR models, conditional and unconditional, seem to perform the better than other models.

When we compare the performance of the models across the four currencies, we observe that the overall the models do not perform very well in case of US Dollar and the Japanese Yen, both failing in seven out of the total fifteen cases. In particular, we find that the best performing model, the Conditional EVT based VaR fails only in the case of USD. It is notable that the VaR models perform poorly against the two managed float currencies, USD and JPY, while performing adequately against the Euro and the Pound. The incidence of failure is twice as high for the managed float currencies as compared to the free float currencies.

5. SUMMARY AND CONCLUSIONS

In the debate over the merits of managed vs. floating exchange rates a key issue that has remained under-examined is the relative efficacy of the risk management tools under the two regimes. This paper attempts to address this question with respect to the Pakistani rupee which seems to be actively managed mainly against the US dollar, but to a lesser extend against other hard currencies. This practice of differentially managing exchange rates provides us with an opportunity to study the implications of the exchange regime for implementing risk assessment models.

Our main focus is on the Value-at-Risk (VaR) model which has been widely adopted as a way of monitoring and managing market risk. In particular, it has been specified by the Bank for International Settlement (BIS) as well as by many central banks as a basis for setting regulatory minimum capital standards. We use five variations of the VaR model, including models based on the Extreme Value Theory (EVT). We find that the exchange rate returns distributions are fat tailed and well-suited for the application of EVT based models. However, we also find that the distributional characteristics are quite different for the four currencies; the USD rate which is an actively managed float, in particular, exhibits fat tails indicating low normal volatility but higher extreme volatility.

This finding conforms to the earlier cross-countries research, for example, by Genberg and Swoboda (2004). In addition, we also find that the dynamic processes are remarkably different for the four exchange rates; the principal object of managed float, USD, exhibits notable serial autocorrelation, as opposed to the other currencies.

In assessing the efficacy of the risk models, we find that the models do not perform very well in case of exchange rates within managed float regimes, i.e., US dollar or Japanese Yen. In the first case, the dollar itself is considered as a free-float currency, but the USD/PKR rate seems to be a managed float. The Yen on the other hand is considered being a managed float currency, but the JPY/PKR rate may not be so managed from the Pakistani side. Either way, when either of the currency in the exchange pair is in the managed float regime, it seems to be harder to assess foreign exchange risk, relative to when both of the paired currencies are in market or free float regime. Since the managed float regime would make it more challenging to model and manage exchange rate risk, there are implications for the exchange rate policy-makers. The managed float regime increases the foreign exchange risk for all economic agents, including, for example, the foreign portfolio and direct investors, as well as the importers and exporters. The additional risk and the associated economic cost may substantial inhibit economic transaction involving foreign exchange to the detriment of the country's growth.

The implementation of the risk models in a developing country like Pakistan poses special challenges. First, the incidence of extreme events and volatility is much higher, since the economic processes may not be stable due to the evolving institutional and regulatory environment. Therefore, the emerging countries exhibit much smaller variations of the nominal exchange rate, yet occasionally experience extreme movement in the exchange rates. As noted in the introductory section, Pakistan has been buffeted by a series of economic shocks. Second, structural economic weaknesses have led to a steady depreciation in the value of rupee. The dilemma for the country is whether to let the currency slide gradually in small increments or to try to maintain a stable exchange rate until realignment becomes inevitable. Under a managed float the risk arising out of short-term volatility is reduced, but at the expense of increasing risk originating from extreme rate movements.

Our analysis provides a framework for jointly considering the two sources of risk. Our results indicate that a market based exchange rates regime will reduce the overall foreign exchange risk. Pakistan has historically followed different exchange rate regimes. Currently, though the country is categorised as managed floaters, its *de facto* operating regime is different from its *de jure* regime which is described as managing *ad hoc* adjustable pegs; Rajan (2012). The findings of this paper provide additional support for following a free float policy in practice as well as officially stated.

Our findings also have direct implications for the operationalisation of risk models, and underscore the importance of correctly specifying the return distributions as well as the dynamic process. Our back-testing exercise shows that the VaR measures with dynamic adjustment for volatility clustering perform better than measures which are based on normal distribution assumption, or which do not take the dynamics of volatility into account, and indicates that the exchange rate risk is better modeled using the Extreme Value Theory. However, we find that the distributional characteristics and volatility structure of exchange rates are different in case of different currencies. The

study suggests that the static extreme loss estimates based on one period may not be a reliable guide to the risk of actual losses during subsequent periods, and need to be updated using a dynamic framework. This finding underscores the fundament problem of dealing with uncertainty, i.e., dealing with the model risk arising from incorrect model specification. Moreover, the parameters of the empirical distribution may also unexpectedly shift in times of financial turbulence and may render models of risk assessment unhelpful. A dynamic VaR based system can be more adaptive to the changing markets conditions and the losses are likely to be less severe than in static risk measurement system.

APPENDIX A

VALUE AT RISK AND THE EXTREME VALUE THEORY

1. Dynamic Value-at-Risk

Following the methodology suggested by McNeil and Frey (2000), we incorporate the conditional volatility, the GARCH effects, as follows. Let ($X_t \; t \in \mathbb{Z}$) be a stationary time series representing the daily observations of a log-return of financial asset price. We assume that dynamics of *X* are given by:

$$X_t = \mu_t + \sigma_t Z_t, \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Where μ_t and σ_t measures the mean return and volatility of the process respectively, Z_t are the innovations which is strict white noise process with zero mean, unit variance and marginal distribution function $F_Z(z)$. We assume that μ_t and σ_t are measurable with respect to $\mathcal{G}_{t-1}$ .Let $F_X(x)$ denote the marginal distribution of stationary time series (X_t) and let $F_{(X_{t+1}\,|\,\mathcal{G}_t)}(x)$ denote the 1-step predictive distribution of the returns over the next day, given knowledge of returns up to and including day *t*.

The mean returns and the volatility of the GARCH (1,1) model with normal innovations has the following specification:

$$\mu_t = \mu \text{ and } \sigma^2_t = w + \alpha(X_{t-1} - \mu)^2 + \beta\, \sigma^2_{t-1}$$

with *w*, α, β>0, and α + β <1. Similarly the mean returns and the volatility of AR(2)-GARCH(1,1) model is:

$$\mu_t = \varphi_1 X_{t-1} + \varphi_2 X_{t-2} \text{ and } \sigma^2_t = w + \alpha(X_{t-1} - \mu_{t-1})^2 + \beta\, \sigma^2_{t-1}$$

The stochastic variable Z_t may be assumed to follows the Normal distribution, or alternatively a *t*-distribution where $Z_t = \epsilon_t \sqrt{\frac{\vartheta - 2}{\vartheta}}$ and ϵ_t follows a Student-*t* distribution with $\vartheta > 2$ degrees of freedom.

We're then interested in estimating quantiles in the tails of these distributions. For $0 < q < 1$, a conditional quantile is a quantile of the predictive distribution for the return over the next day denoted by:

$$x_q^t = \inf\{x \in \mathbb{R}: F_{(X_{t+1}\,|\,\mathcal{G}_t)}(x) \geq q\}\,, \quad \text{where}$$

$$F_{(X_{t+1}\,|\,\mathcal{G}_t)}(x) = P\{\sigma_{t+1} Z_{t+1} + \mu_{t+1} \leq x \,|\, \mathcal{G}_t\} = F_z((x - \mu_{t+1})/\sigma_{t+1})$$

which implies,

$$x_q^t = \mu_{t+1} + \sigma_{t+1} z_q \,, \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

where z_q is the upper qth quantile of the marginal distribution of innovation distribution which does not depend on *t*. The next step is to model the tails of the innovation distribution Z_t using Extreme Value Theory.

2. Extreme Value Theory (EVT) Models of Distribution Tails

According to EVT, the form of the distribution of extreme returns is precisely known and independent of the process generating returns; see for example, Longin (1996), Longin and Solnik (2001) and Chou (2005), and Diebold, *et al*. (2000) for a note of caution. The family of extreme value distributions can be presented under a single parameterisation, known as the Generalised Extreme Value (GEV) distribution. There are two ways of modeling extremes of a variable. One approach is to subdivide the sample into *m* blocks and then obtain the maximum from each block, the *block maxima method*. The distribution of block maxima can be modeled by fitting the GEV to the set of block maxima. An alternative approach takes large values of the sample which exceed a certain threshold *u,* the peak-over-threshold (POT) approach. The distribution function of these *exceedances* is then obtained employing fat-tailed distributions models such as the Generalised Pareto Distribution (GPD). However, the POT approach is the preferred approach in modeling financial time series.

Fisher and Tippett (1928) developed the theory describing the limiting distribution of sample maxima and the distribution of *exceedances* above a threshold. Building on their work, Pickands (1975), Balkema and de Haan (1974) state the following theorem regarding the conditional excess distribution function.

Theorem: For a large class of underlying distribution functions the conditional excess distribution function F_u (y), for a large value of μ, is well approximated by:

$$F_\mu(y) \approxeq G_{\beta,\xi}(y) \; ; \; \mu \to \infty$$
$$G_{\beta,\xi}(y) = 1 - (1 + \xi y/\beta)^{-1/\xi} \,, \; \xi \neq 0$$
$$= 1 - e^{-y/\beta}, \; \xi = 0$$

for $y \in [0, x_F - \mu]$ if $\xi > 0$, and $y \in [0, -\beta/\xi]$ if $\xi < 0$. $y = (x - \mu)$ and μ is the threshold; $x_F \leq \infty$ is the right endpoint of F. $G_{\beta,\xi}(y)$ is known as the Generalised Pareto Distribution (GPD). $F_\mu(y)$ can also be reformulated in terms of F(x) describing the entire time series X_t to construct a tail estimator for the underlying distribution. The important step in this procedure is to determine the threshold for identifying the tail region. It involves a trade-off: a very high threshold level may provide too few points for estimation, while a low threshold level may render a poor approximation. Several researchers, [e.g., McNeil (1997, 1999)] suggest employing a high enough percentile as the threshold. However, we consider Mean excess function plot in this regard.

Using as an estimator of F(u) the ratio $(n - N_u)/n$, where *n* is the total number of observations and N_u is the number of observations above the threshold, the tail estimator is defined as:

$$F(x) = 1 - N_u/n(1 + \xi(x-\mu)/\beta)^{-1/\xi}$$

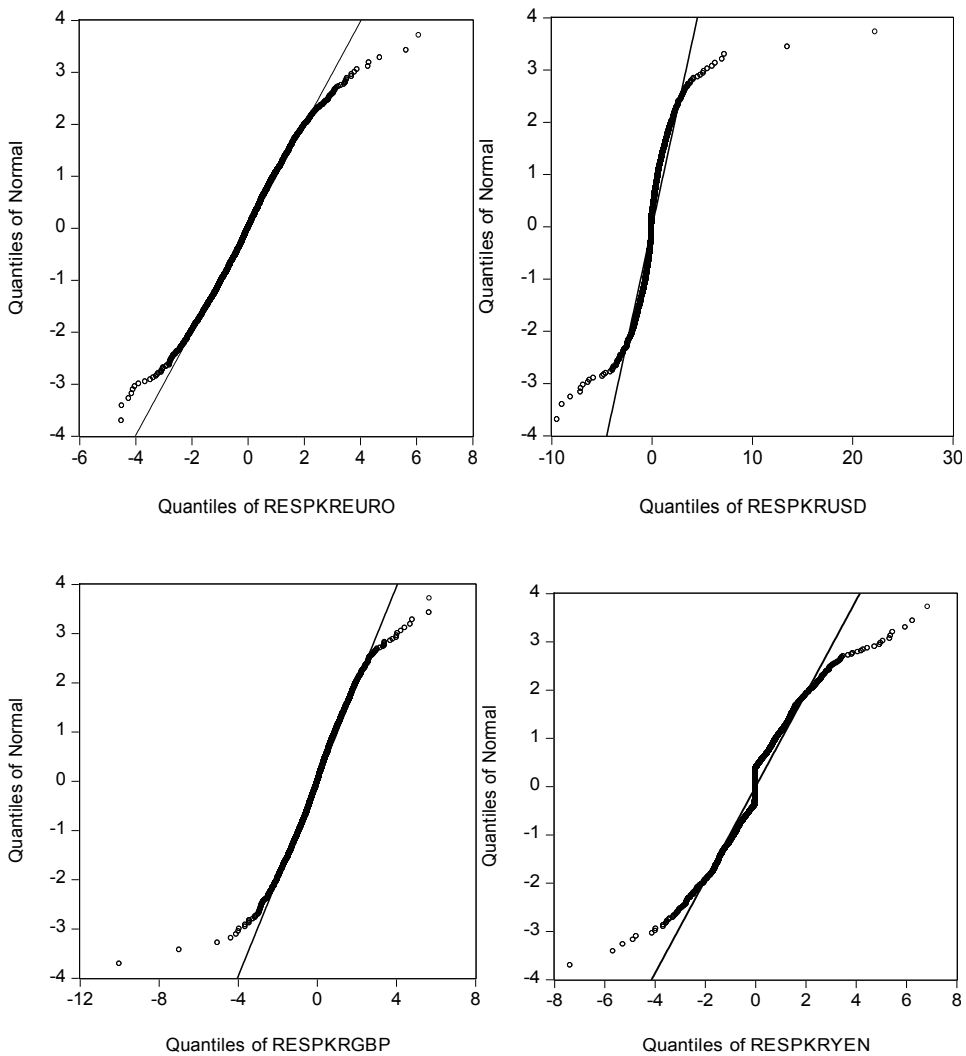
for $x>u$. For a given probability, $q>F(u)$, the VaR estimate is obtained by inverting the tail estimation formula above to get [see Embrechts, *et al.* (1997)].

$$z_q = \mathrm{VaR}_q = \mu + \beta/\xi\,((n/N_u(1-q))^{-\xi} - 1).$$

The estimation of the GPD parameters, ξ and β is made using the method of maximum likelihood. Finally the estimated dynamic or conditional VaR using Eq. (1) is:

$$\hat{x}_q^t = \widehat{\mu_{t+1}} + \hat{z}_q \widehat{\sigma_{t+1}}. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

APPENDIX - B: Q-Q PLOTS

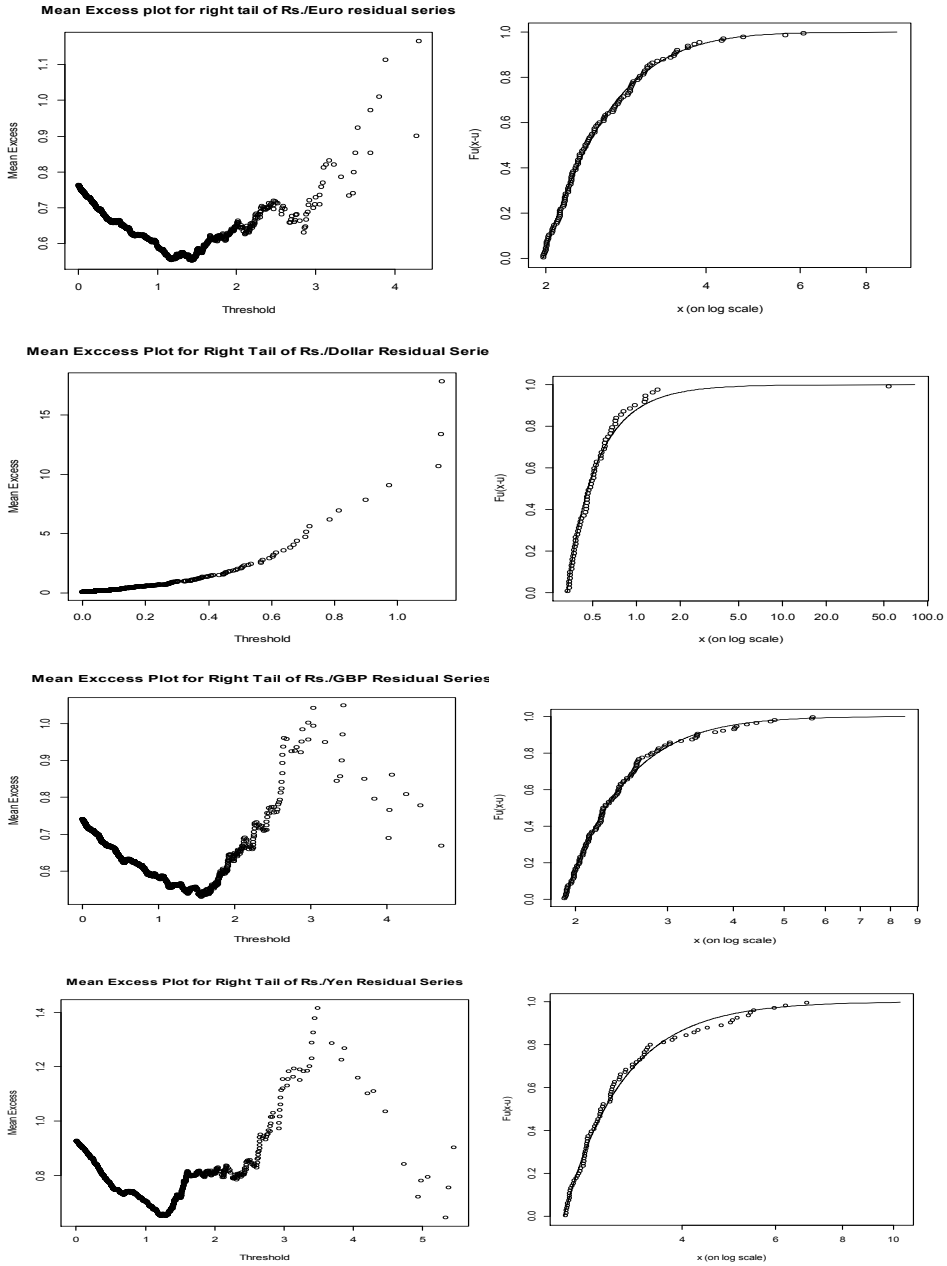


The figure shows Q-Q plot against normal distribution of the residuals series for EUR, USD, GBP and JPY respectively.

APPENDIX C

MEAN EXCESS AND EXCESS DISTIBUTION PLOTS

Fig. 4. The left figure shows Mean excess function (ME) plotted against the threshold for right tail, whereas the right figure shows excess distribution plot for the goodness-of-fit of each residual series.



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